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A BAYESIAN NETWORK APPROACH TO IMPROVE CHANGE PROPAGATION ANALYSIS

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Abstract

Predicting and analysis of change propagation is one of the important issues in engineering change management. In this paper, we illustrate how the Bayesian network (BN), which is emerging tool for a wide range of risk management, can be used in change propagation analysis. The paper compares BN with most popular tool for change propagation analysis, namely CPM. Firstly, we show that CPM can be directly converted into an equivalent BN. In addition to this, we also show that BN has significant advantages over CPM at both modeling and analysis level. In the model-ing level, various assumption over CPM can be relaxed and various kinds of modeling exten-sion can be accommodated with BN. At the analysis level, BN's ability to performing probabilistic inference provides a user with another interesting measures, which cannot be obtained with CPM. Moreover, BN provides robust framework for learning change propagation probabilities from empirical data. Our method can enhance the capability of engineering change management throughout entire product life-cycle.

Keywords: Change management, Change propagation, Bayesian Network, Design Management, Design informatics

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1 INTRODUCTION

The difficulty of managing changes lies in the fact that a change does not occur alone. In complex system where each part is associated with different parts, a change presented into a part may influence other parts. As a result, a change may propagate throughout entire system. The change propagation process shows uncertain behaviour (Eckert et al., 2004); it might be terminated within a few steps, but sometimes it generates an avalanche of changes, increasing the number of changes as the redesign progress. To cope with such uncertainty, a number of strategy is proposed in order to avoid changes as much as possible (Prasad, 1996). These strategies includes for example to make changes as early as possible, or to create design buffers. However, eliminate engineering changes is both undesirable and unrealistic.

One way to improve change management is to anticipate the consequence of changes in advance by which the design task can be guided (Clarkson et al., 2004). There has been some literature which proposes models or tools for predicting change propagation process of the system. One of the first is Cohen et al. (2000) which proposes a data model which can anticipate the change propagation process, named as Change Favourable Representation (C-FAR). C-FAR represents design entities as vectors and its design parameter as components of a vector. A matrix, named as C-FAR matrix is used to construct the parameter linkage between different entities. By multi-plying C-FAR matrix, a consequence of a change from a source entity to a target entity can be calculated. Cheng and Chu (2012) utilizes network theory in analysing change impacts on each component. By representing the product as complex network which consist of nodes and arcs, it provides a user with some metrics (degree-changeability, reach-changeability and be-tween-changeability) which could enhance the change propagation analysis. However, the above literature has limitation because they do not address the uncertain aspect of the change propagation process.

Clarkson et al. (2004) propose change propagation methods (CPM). Distinctive feature of the CPM is that it tries to address the uncertainty of the change propagation process by measuring risk of each component. In the CPM, the risk is obtained by multiplication of likelihood and impact. A design structure matrix (DSM) is utilized for quantifying likelihood and impact be-tween adjacent components. After then, an algorithm calculates the expected risk of each component by enumerating every possible change propagation path derived from DSM. After its successful implementation, CPM has been extended by numerous literature. For example, extending DSM into MDM (multi domain matrix), Koh et al. (2012) addresses change propagation among different domains such as organization, or manufacturing process. Keller et al. (2005) proposes visualization tool for utilizing various information which can be obtained by CPM.

Bayesian network (BN) is a complete representation about probabilistic relationship among (discrete or continuous) random variables (Jensen, 1996). Due to its ability to compactly representing dependence and independence relationship among random variables, it has been used as robust and efficient framework for modelling and reasoning uncertain knowledge which is often represented by large number of variables (Kjræulff and Madsen, 2012). After its concept was first introduced in the field of artificial intelligence, it has been utilized in various real world application including medical diagnosis, document classification, bio-informatics, and fault detection of complex systems (Heckerman et al., 1995a).

The objective of this paper is to explore the capabilities of the BN formalism in modelling and analysis of change propagation process. The paper compares BN with most popular tool for change propagation analysis, namely CPM. Firstly, we show that CPM can be directly converted into an equivalent BN. In addition to this, we also show that BN has significant advantages over CPM at both modelling and analysis level. In the modelling level, various assumption over CPM can be relaxed and various kinds of modelling extension can be accommodated with BN. At the analysis level, BN's ability to performing probabilistic inference provides a user with another interesting measures, which cannot be obtained with CPM. Moreover, BN provides robust framework for learning change propagation probabilities from empirical data.

2 CHANGE PROPAGATION MODELING USING BAYESIAN NETWORK

2.1 Change Prediction Methods

Among numerous change prediction methodologies, one of the most established approach is change propagation methods (CPM) proposed by Clarkson et al. (2004). Distinctive feature of the CPM is that it adopts the concept of likelihood and impact in order to provide risk measures of change propagation. CPM is constructed as following steps:

- Direct connection between components are identified throughout design structure matrix (DSM). When the value of $(i, j)^{th}$ element in DSM is 1, it indicates existence of change propagation path from component i to j, and 0 otherwise.
- For each connection identified in previous steps, likelihood and impact of change propagation is identified. Both values are estimated based on prior experience or knowledge of the experts. For example, the change likelihood in $(i, j)^{th}$ element in the DSM represent the probability that component i's change cause component j's change. On the other hands, the change impact identified in $(i, j)^{th}$ element in the matrix represent the design effort that would be required to modify j when component i has been changed.
- The combined risk of change propagation is calculated using algorithm proposed by XXX. After then, an algorithm calculates the expected risk of each component by enumerating every possible change propagation path derived from DSM. For the detail of this algorithm, referred to Clarkson et al. (2004).

However, CPM suffers from several limitations. First, All the likelihood and impact values within DSM should be solely depends on engineer's opinion. When the size of the product becomes larger, this may be problematic. Although Lee et al. (2009) tries to enhance this subjectivity problem using analytical network process (ANP), it cannot avoid the limitation of the expert elicitation. Second, CPM only focused on binary relationship between components. However, one design change can impact more than two components simultaneously and vice versa. Since the DSM cannot encodes the relationship more than three components simultaneously, the model cannot accurately model the behaviour of change propagation process. Finally, flexible querying about risk analysis is impossible. Since CPM only focused on the deriving the risk index of individual component. In order to tackle these limitations the concept of Bayesian Network and its application is described in following subsections.

2.2 Bayesian Network

A BN is graphical model for representing and reasoning about a set of random variables. A BN is consist of both qualitative and quantitative part. The qualitative part is represented by the graph. The nodes in the graph represents a set of random variables, $X = \{X_1, X_2, ..., X_n\}$, from a domain. A set of directed edge (or arc) connecting pairs of node $X_i \rightarrow X_j$, represents direct dependency between random variables, indicating X_i is the direct cause of X_j. The only constraint on the arcs is that it should not make any directed cycle in the graph. Therefore, whenever the directed acyclic graph (DAG) assumption is maintained, any kind of cause-effect relationship among random variable can be encoded with on the BN.

For a quantitative part, the strength of the relationship between variable is quantified by conditional probability tables (CPT). Assuming discrete state of each variable, the CPT contains a set of conditional probability distributions given every possible instantiation of its parents. An example of CPT of simple BN is illustrated in Figure 1. Assuming every variable has binary state, CPT of C consists of four distinctive conditional probability distribution given the combination of A and B. On the other hands, for A and B, which has no parents, are specified with prior distribution given no conditional term. These variables having no parents are called root variables.

BN utilizes conditional independent statements encoded on the variables in reducing the burden of calculating joint probability distribution. Conditional independence statements can be read off by the d-separation (and the equivalent directed, global Markov property) which is originated from the topology of the graph. The d-seperation property encoded in the graph enables the full joint probability distribution of entire random variables $X = \{X_1, X_2, ..., X_n\}$ to be factorized as in following equation (1), which is often called chain-rule:

$$P(X_1, X_2, ..., X_n) = \prod_{i=1}^{n} P(X_i \mid X_{pa(i)}) \quad (1)$$

For example, the full joint probability distribution of four random variables in Figure 1, can be obtained by the product of only four different probability distribution as in equation (2)

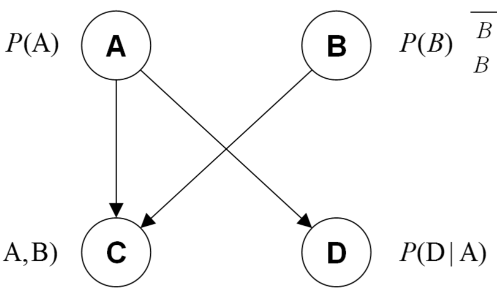
$$P(X_A, X_B, X_C, X_D) = P(A)P(B)\mathrm{P(C \mid A,B)}\mathrm{P(D \mid A)} \quad (2)$$

In this way, BN could efficiently reduce computational burden of calculating full joint probability distribution, which grows exponentially with the number of random variables. For the detail about the concept of d-seperation and conditional independence, consult with Pearl (1988).

After modelling a BN, it can be used to answer various types of probabilistic queries about them. BN utilizes inference algorithm for exploiting conditional independence assumption encoded in them in order to make this calculation efficiently. However, when the number of nodes increase, the exact inference task on BN becomes a NP-hard problem. Fortunately, efficient inference algorithm have been developed such that inference task can be done within a second even for complex network consists of more than hundreds of random variables. However, the detailed description of the inference algorithm is beyond the scope of this paper because there are many well-established commercially available tool which can support automatic calculation of posterior probability distribution. (the interested reader might be referred to Pearl (1988)).

$A = true$	0.3
$A = false$	0.7

$B = true$	0.5
$B = false$	0.5



	$A = true$		$A = false$	
	$B = true$	$B = false$	$B = true$	$B = false$
$C = true$	0.9	0.1	0.95	0.12
$C = false$	0.1	0.9	0.05	0.88

	$A = true$	$A = false$
$D = true$	0.8	0.3
$D = false$	0.2	0.7

Figure 1. An example of a BN with four random variables

2.3 Modelling Change Propagation Using Dynamic Bayesian Network

Dynamic Bayesian network (DBN) (Murphy, 2002) which is special types of BN can be utilized for modelling change propagation. It describes a system that is dynamically changing over time by modelling stochastic variables over time. It is consist of a sequence of sub-models, each of which represents the state of the system at a certain point in times, or namely the time-slice. Temporal edges connecting nodes between consecutive sub-models reveals these temporal dependency between states. Although the DBN models the dynamic system, its structure is time-invariant; the structure of the network does not change over time with the exception of the root node (i.e. nodes at initial time-slice). Therefore, whenever the prior distribution of root node is specified, the DBN can recursively update the network, enabling a user to predict the further behaviour of the system for the desired number of times.

Change propagation process can be naturally translated into an equivalent DBN. In Figure 2, a simple DSM is converted into equivalent BN. In this network, each time slice consists of a set of nodes c_i^t which represents the state of each component i at time t. This node is discrete and binary random variable which takes values 'yes' if the corresponding component changes and 'no' otherwise. Now, the temporal edges connecting two nodes between consecutive time-slice $c_i^{t-1} \to c_j^t$, reveals direct dependency from component i to j. Temporal edges can be easily identified by referring (i, j) the cell in the DSM. Now, a user can unroll the dynamic model for the desired number of time steps, duplicating a set of nodes and temporal edges.

Now, quantitative strength between components can be specified by the CPT. Because the state of component at time t only depends on the components at previous stages, parameters of each CPT is determined associated with every combination of the state of its affecting components at time $t-1$. Suppose, for example, that the component j is only affected by the component i. If we denote $l_{i,j}$ to the likelihood that the change is propagated from component i to j, which is defined on the DSM, the CPT of c_j^t can be defined as follows:

$$\begin{aligned}
&p(c_j^t = yes \mid c_i^{t-1} = yes) = l_{i,j} \\
&p(c_j^t = no \mid c_i^{t-1} = yes) = 1 - l_{i,j} \\
&p(c_j^t = yes \mid c_i^{t-1} = no) = 0 \\
&p(c_j^t = no \mid c_i^{t-1} = no) = 1
\end{aligned} \tag{3}$$

Although the component has multiple predecessors, its conditional probability distribution can be generalized with noisy-OR gate. Noisy-OR gate are applied when there are several causes $X_1, X_2, \ldots, X_n$ and a common effect variable Y, where (1) each of the cause X_i has probability p_i of being sufficient to produce effect, and (2) the ability of each cause being sufficient to produce effect is independent of the presence of other causes, i.e. causally independent with each other. In this case, the probability of y given an instantiation of its parent X_p is given by following equation:

$$p(y \mid X_p) = 1 - \prod_{i:X_i \in X_p} \left(1 - p_i\right) \tag{4}$$

As in above equation, the complexity of conditional probability distribution is reduced from exponential to linear in the number of parents. It is noteworthy, however, that the Noisy-OR gate is a special case of conditional probability distribution. Relaxing assumption of the Noisy-OR gate, flexible generalization of probability relationship is possible. We will address this issue in further sections.

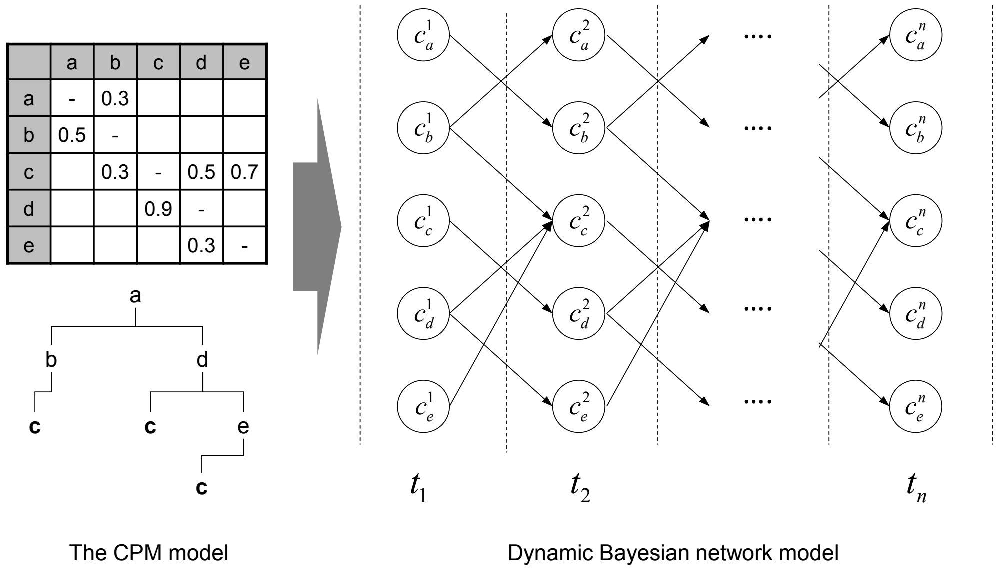


Figure 2. Conversion from the DSM into BN

3 BENEFITS OF USING BN

So far, we have shown that change propagation process can be naturally represented with BN. However, utilizing the modelling aspects underlying BN, more flexible generalization of the original model is possible. In later sub-sections, we will illustrate a number of modelling extensions and show how they are represented with BN.

3.1 Improvement in modelling

BN provides more powerful representation scheme than previous model. In the CPM, only the interaction between two components are used as input for deriving combined likelihood. In previous chapter, we have shown that an equivalent BN has the CPT constructed with the Noisy-OR gate. However, BN has no restriction on how parents interact with a child. Thus, any numbers can be used for constructing CPT whenever it is compatible with the graph structure. This enable a modeller to specify the interaction among more than three component at the same time.

For example, consider CPT of component c in table 1, which is affected by component a and b. The probability of component c's having changes given both a and b have changed is computed as $1-(1-0.3)(1-0.5)=0.85$ by if we follow Noisy-OR gate. This calculation is based on the assumption that both d and e interact independently with c. However, BN can relax such independence assumption by manually specifying any numbers in the CPT. If such dependence is assumed, the probabilistic impacts to a common child b can be amplified (by having more than 0.72) or abbreviated (less than 0.72) in the presence of the change of both parents. In this way, user can represent more nuanced interaction among components, even when more than three components interact simultaneously.

Table 1. Conditional probability table of component c (based on Figure 1.)

Component a	Component b	Yes	No
Yes	yes	1-(1-0.3)(1-0.5) = 0.85	(1-0.3)(1-0.5) = 0.15
Yes	no	1-(1-0.3) = 0.5	(1-0.3) = 0.5
No	yes	1-(1-0.3) = 0.3	(1-0.3) = 0.7
No	no	0	1

Moreover, BN can represent hierarchical structure of the product. When hierarchical structure of the product is assumed, a change presented in a part or system may propagate across different hierarchical levels (Eckert et al., 2004). An example is illustrated in left panel in the Figure 2. At the system level a change of engine might require generic changes in bare fuselage. However, the changes in engine system also can affect the casing, the subpart of bare fuselage. Moreover, within a system, changes in part level also can lead to changes in upper system. In this situation, it might be helpful to analyse change propagation process with varying degree of hierarchical levels. By introducing additional random variables, BN can easily accommodate hierarchical structure in the model. For example, the above example can be converted into an equivalent BN illustrated in Figure 2. This BN consists of two additional nodes s_1 and s_2 which represents each subsystem components. Connecting edges within each time-slice (solid line) represents interaction within a system. By specifying CPT of each system level, we can represent probabilistic relationship among system and its subparts. Temporal edges connecting between adjacent time-slice (dotted line) represents interaction between different systems. As illustrated in the figure, changes can propagate from system to component, component to system, or system to system.

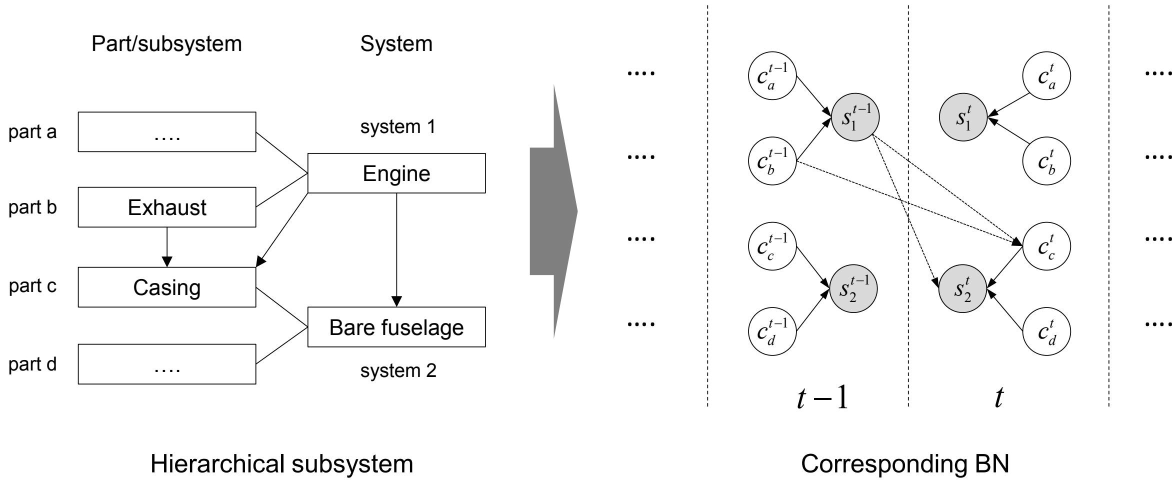


Figure 3. Modelling change propagation between parts and subsystem

Finally, BN can represent multiple types of interaction among components. Although CPM assumes unified interface between components, actually there are various types of 'linking parameters' between parts and subsystems such as geometry, force, heat, electronic or material, etc (Eckert et al., 2004), which is illustrated Figure 4. BN can easily accommodate dependency between different parameters with the form of CPT. For example, interaction between engines and bearings can be specified via CPT as illustrated in Table 2. If engines geometry parameter might affects the bearings mechanical vibrations, this relationship can be quantified via conditional probability $p(\text{bearing} = \textit{mech.vibration} \mid \text{engine} = \textit{geometry})$. In this way, one can reveal the interfaces among components in more detailed manner. This property can also be applied to model change routing algorithm of which behaviour is described in Giffin et al. (2009)'s case study. They shows that that considerable amount of engineering change orders are rejected as well as reflected to other components. In this sense, an engineer may need to evaluate the change request and determine whether or not to implement the request. BN can describe this change rejection/adoption mechanism by introducing another state 'changes reflection' can lies between 'change' and 'not changes'

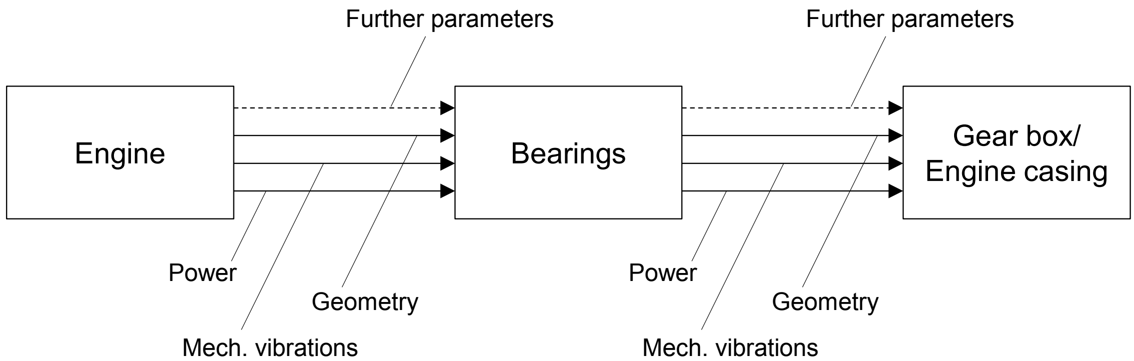


Figure 4. multi-level parameter linkage

Table 2. an example of CPT when multi-level linkage is assumed

Engine	Bearing			
	Power	Mech. vibrations	Geometry	Further parameters
Power	0.7	0.2	0.1	0
Mech. vibrations	0.1	0.8	0.05	0.05
Geometry	0.3	0.3	0.3	0.1
Further parameters	0	0	0.5	0.5

3.2 Improvement in analysis

Another benefit of using BN is that it enables a user to conduct more various types of analyses than CPM does. This can be possible via probabilistic inference, which estimates the conditional probability of each nodes given specified values of other nodes. In probabilistic inference, a user firstly specify the values of any combination of nodes in the network they have observed, which is referred as evidence e. This e, then, propagates across the network, updating a new posterior probability distribution $p(X \mid e)$ for each variable in the network.

Using this concept, probabilistic inference can be used to simulate the state of each component given the change of other component. Most common types of inference we wish to perform might be to find out updated posterior probability of components given the change of the component at initial stages. In the BN, this process of querying about the state of the system at future time given current evidence is referred to predictive inference. By predicting the future state of component given the change of current component, we can expect similar result with CPM.

However, BN has advantages over CPM because it can perform different types of inference supporting any direction of reasoning. Contrary to predictive inference, inference from symptoms (affected component) to cause (affecting component) is also possible. This types of reasoning is often called diagnostic inference. Diagnostic inference provides an analyser with information about relative criticality of each component given a change of specific component. In addition to this, one can reason about the mutual effect of a common cause. For example, an analyser might need to identify the pairs of components which are changed together. Probabilistic inference can also be used to identify such mutual relationship among affected components by querying affected components.

3.3 Improvement in Data Learning

One of the distinctive advantages using BN is that it has ability to learning from the data. In most of change management system, there are bunch of data which can be used for estimating probabilities between components. For example, most of change management systems contains change logs which contains historical records in terms of what was changed, why it was changed or who changed it, etc. Utilizing these data, BN provides intuitive way to learning probabilities between components based on the combination of experts' opinion and empirical data.

The basic idea of learning probabilities is similar to the standard statistical modelling approach. Although the distribution of the real data is unknown, we assume that it belongs to the specific family of the possible distribution. We can assume that each probability within CPT belongs to the distinct members of this distribution family, which is represented by the parameter θ. This contrast to the previous model where each parameter is specified precisely. When new data is available, our task is to find optimal parameter θ which provides most probable explanation about the current data set.

The overall procedure is illustrated in Figure 5. Firstly, an analyser expresses the uncertainty about the parameters with the form of prior distribution. This implies that an expert is not only asked with most likely values (means), but also their confidence about these values represented with error terms (variance). When the project begins, we can obtain dependency data from a set of engineering change logs. These statistical data is combined with prior distribution and posterior distribution is calculated according to the Bayes' theorem. Now the posterior parameters become priors for the next data set. In this way, the model continuously adapt their optimal parameters in light of the new data. As more data is utilized in parameter learning, more accurate parameter can be obtained.

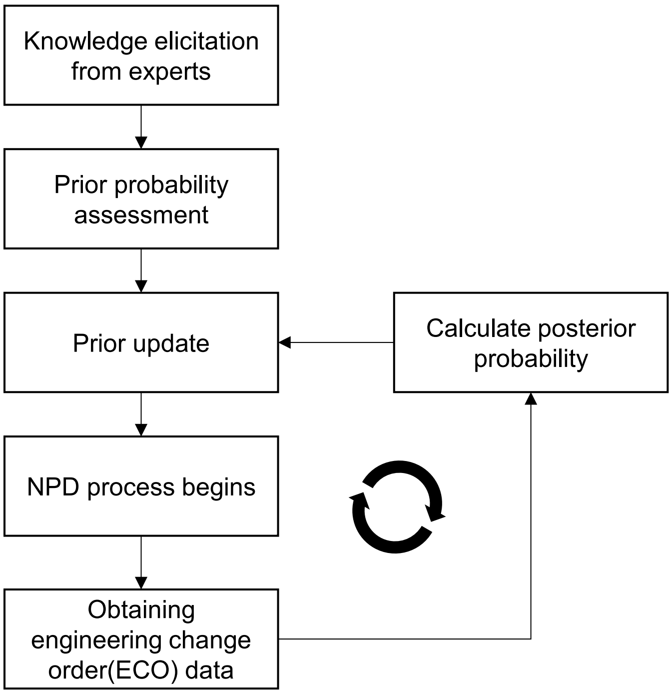


Figure 5. Parameter learning procedure of BN

In Bayesian updating of parameters, conjugate prior is used as convenient approximation to facilitate analysis. To construct Beta prior of CPT, an expert is asked for not only estimating mean values about change probabilities but also standard deviation around the means indicating uncertainty about his estimation. The specified mean and standard deviation, then determines the shape parameters α_y and α_n of the Beta distribution. These parameters can be obtained by solving following equations:

$$\begin{aligned} c_y : E(\theta_y) &= \frac{\alpha_y}{\alpha_y + \alpha_n} \quad \text{and} \quad Var(\theta_y) = \frac{\alpha_y \alpha_n}{(\alpha_y + \alpha_n)^2 (\alpha_y + \alpha_n + 1)} \\ c_n : E(\theta_y) &= \frac{\alpha_n}{\alpha_y + \alpha_n} \quad \text{and} \quad Var(\theta_y) = \frac{\alpha_y \alpha_n}{(\alpha_y + \alpha_n)^2 (\alpha_y + \alpha_n + 1)} \end{aligned} \quad . \tag{5}$$

We can regard α_y and α_n as imaginary numbers which represent a case of changes and not changes respectively. Sum of shape parameters $\alpha_y + \alpha_n$ is often called equivalent sample size, which can be thought of as imaginary counts from our prior experience. Therefore, the larger the equivalent sample size, the more confident we are in our prior

Table 3 shows an example of Beta prior of CPT based on the assessment made in Table 1. Given four combination of its parents a and b, an expert is asked for specifying four independent Beta distribution. Contrary to the point assessment, an expert should express their precision in their mean values. After observing a data, shape parameters are updated following the Bayes' theorem. Updating procedure is simple; If we denote y to the number of case corresponding to α_y and x to α_x, then prior distribution with shape parameter $\text{Be}(\alpha_y,\ \alpha_n)$ is updated to posterior distribution $\text{Be}(\alpha_y + y,\ \alpha_n + n)$. Table 3 also illustrates the results of updating posterior probability distribution shown in Table 3 in the light of some hypothetical data. Now, the prior distribution $\text{Be}(1.8954,\ 0.3281)$ is updated to a $\text{Be}(1.8954 + 2,\ 0.3281 + 1)$ posterior distribution which has mean value of changes 0.7457, and standard deviation 0.1754. The results obtained are fairly intuitive: by observing two out of three cases of changes, prior probability of component change is reduced from 0.85 to 0.7457. Moreover, by observing additional cases, precision about the mean value is also reduced from 0.2 to 0.1754. Similar updates can be performed simultaneously for all nodes in the network without much cost to perform the analysis.

Table 4. Beta prior CPT with expressed mean and standard deviation

Component d	Component e	shape parameters (prior) α_y	α_n	hypothetical data yes	no	shape parameters (posterior) α_y	α_n	mean (change)	std. (change)
yes	yes	1.8954	0.3281	2	1	3.8954	1.3281	0.7457	0.1745
yes	no	12	12	5	7	17	19	0.4722	0.0821
no	yes	6	14	8	15	14	29	0.3256	0.0706
no	no	-	-	-	-	-	-	-	-

4 CONCLUSION AND FUTURE WORKS

In this paper we have shown that the applicability of the BN in change propagation modelling and analysis. The main advantages of using BN is the ability to incorporate expert opinion in more flexible manner. Because BN has no restriction in defining structure of the network or specifying CPT of each node, it can be applicable to more generic situation, which make it useful tool in practice. At the analysis level, BN can be used to simulate various change scenarios to help analyser in managing changes. In this paper, we have shown that various types of probabilistic inference such as predictive, diagnostic, or inter-causal inference, can generate unusual measures which cannot be obtained with CPM. Finally, one positive aspect using BN is that it provides mathematically rigor framework for updating change propagation probabilities in light of some observed data. Based on the Bayes' theorem, we have shown that the BN can continuously adapt the model in light of some observed data. However, there are also challenges to overcome in applying the BN to the change propagation model. One of such challenge is that the model become too complex with many nodes to specify, which requires a significant effort of domain expert. Especially for a complex product which consists of a large numbers of systems, it might be impossible to even specify the structure of the graph. This challenges address interesting future research toward structural learning from the data. Actually, developing an algorithm for inferring both the network structure and its CPT from empirical data is one of the important issues in BN. The author currently working on developing an efficient algorithm for inferring BN from the actual change log.

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