NordDesign 2014
August 27 – 29, 2014
Espoo, Finland / Melbourne, Australia

A case study of efficient tolerance synthesis in product assemblies under loading

Maciej Mazur, Martin Leary, Aleksandar Subic

RMIT Centre for Additive Manufacturing (CAM)
RMIT University, School of Aerospace, Mechanical and Manufacturing Engineering
maciej.mazur@rmit.edu.au
martin.leary@rmit.edu.au

Abstract

Modelling the effects of loads on mechanical assemblies in statistical tolerance analysis typically requires the use of computationally demanding numerical simulations. The associated Uncertainty Quantification (UQ) methods used for estimating yield are typically based on robust, yet computationally inefficient Monte Carlo (MC) simulation. Identifying optimum tolerances with tolerance synthesis requires multiple iterations of tolerance analysis. When combined with expensive numerical simulation of loading effects and a large number of model evaluation required by MC simulation, the computational cost can increase beyond practical limits.

However, Polynomial Chaos Expansion (PCE) UQ methods have been under recent development which offer higher efficiency than MC sampling. PCE has been recently implemented in a Process Integration and Design Optimization (PIDO) tolerance synthesis approach for assemblies subject to loading. This work reports on an industry-based tolerance synthesis case study which demonstrates the high computational cost reductions achievable with the developed PCE based PIDO approach. The demonstrated approach can be adopted to effectively increase product robustness and manufacturing efficiency.

Keywords: Variation, uncertainty, robustness

1 Introduction

Manufacturing variation results in uncertainty in the functionality and performance of mechanical assemblies. The study of the effects of variation on the functionality of a product part or assembly is known as tolerance analysis. Tolerance synthesis is the process of optimally allocating tolerances to maximize yield and minimize associated costs. The functionality of many mechanical assemblies is dependent on loading through effects such as compliance and dynamics [1]. Accommodating loading in tolerance synthesis allows for a more realistic prediction of assembly behaviour in the presence of manufacturing variation [2]. By addressing the effects of loading and manufacturing variation early in the design phase where cost commitments are low (Figure 1), the cost penalties associated with managing poor quality later in the manufacturing stage when the ability to enact change is limited can be avoided [3, 4].

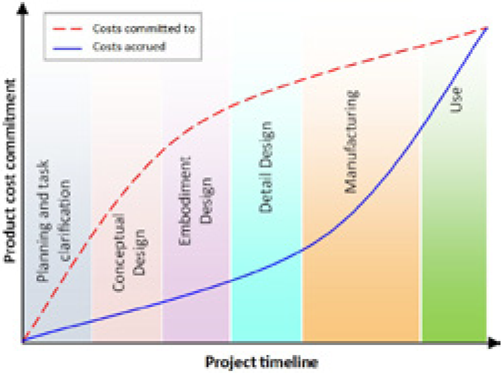

Figure 1. Cost commitment and accruement during design process phases, after [5]

Estimating the effects of variation on assembly functionality is typically achieved with tolerance analysis based on Computer Aided Tolerancing (CAT) models [6]. A number of methods and CAT software tools have been proposed for addressing tolerance analysis and synthesis problems in complex mechanical assemblies. However, current approaches are limited in their ability to comprehensively accommodate tolerance analysis problems in which assembly functionality is dependent on the effects of loading. Limitations include [2, 7-11]:

- Reliance on specific, custom simulation codes with limited implementation in practical and accessible tools, as well as the need for significant additional expertise in formulating specific assembly tolerance models and interpreting results.
- Lack of ability to accommodate assembly loads or accommodation of only specific loading scenarios (such as sheet metal compliance or welding-distortion).

To address these limitations a Process Integration and Design Optimization (PIDO) tool based tolerancing platform has recently been developed [12, 13]. PIDO tools are software frameworks developed for Multi-disciplinary Design Optimization (MDO) which facilitate integration with standalone CAD/E modelling tools for conducting automated parametric analysis, Design of Experiments (DOE) studies, statistical analysis and multi-objective optimization [14-16]. The proposed tolerancing platform is based on the development of scripting links between standalone CAD/E assembly models, and incorporation of the analysis capabilities of PIDO tools. The approach has been demonstrated to effectively address tolerance analysis problems involving assembly loading [13].

Extending the capability of tolerancing tools to accommodate tolerance synthesis, requires that tolerance analysis be iterated, which significantly increases computational costs, especially when numerical modelling of the effects of loading on mechanical assemblies is required. The cost of such tolerance synthesis is often seen as computationally impractical and not warranting the associated benefits [17]. A major reason for the high computational cost is associated with the estimation of yield (i.e. product percentage meeting specification requirements) in tolerance analysis based on Uncertainty Quantification (UQ) methods. The traditional approach to UQ in statistical tolerance analysis is reliant on sampling based UQ methods such as Monte Carlo (MC) sampling. MC sampling is typically applied due to its inherent robustness and broad applicability. However MC sampling has poor efficiency and requires a large number of model evaluations for accurate results, which can impose high computational costs [18].

However a variety of alternative analytical UQ techniques have seen recent development which, under certain conditions, can offer significantly higher efficiency than methods such as MC sampling [7]. A particularly efficient UQ method is Polynomial Chaos Expansion (PCE) [19].

PCE has been recently shown to significantly reduce the computational cost of tolerance analysis and synthesis, and has been integrated into a PIDO based framework for tolerance synthesis in assemblies subject to loading [12]. The PIDO approach integrates: highly efficient PCE UQ; parametric CAD and FE models accommodating the effects of loading; cost-tolerance modelling; yield quantification with Process Capability Indices (PCI); and, optimization of tolerance cost and yield with multi-objective Genetic Algorithm (GA). The tolerance synthesis platform can be applied to tolerance analysis and synthesis with significantly reduced computation time while maintaining accuracy.

This work reports on the application of the previously developed PIDO based tolerance synthesis approach which incorporates PCE [12], to a case study problem involving a product assembly under loading. The product is an automotive rotary switch in which a resistive actuation torque is provided by a spring loaded radial detent acting on the perimeter of the switch body. Functional characteristics require that the resistive switch actuation torque be within an ergonomically desirable range and numerical modelling of loading effects is required to estimate actuation torque. The integration of loading effect modelling is accommodated by the PIDO framework, whereas the associated computation cost of tolerance synthesis are effectively addressed by the application of PCE.

2 Polynomial Chaos Expansion (PCE)

Polynomial Chaos Expansion (PCE) is a method of estimating how input uncertainties in a stochastic system manifest in its outputs, through representation of the system response function using orthogonal polynomial expansions in stochastic variables [20]. The orthogonal polynomials of PCE are based on a Weiner-Askey polynomial scheme, which includes various types of orthogonal polynomials bases that are specifically matched to particular probability distributions of the stochastic variables [19, 21]. A basis is a set of functions whose combination can be used to represent all function in a given function space.

For stochastic variables with a normal distribution (as is common in tolerancing and quality control fields) the orthogonal polynomial basis is formed by the Hermite polynomials [22]. The associated density function is the standard normal (Gaussian) distribution. Weighting functions are applied to the polynomial series basis that are probability density functions describing the stochastic input variables. Other distribution types can be accommodated with different weighting function and corresponding orthogonal polynomial bases [19]. The series is theoretically infinite, however it is truncated in practice. The highest degree of non-truncated polynomial denotes the order of the expansion. The coefficients of the polynomial series may be determined with a number of techniques [23] and once known, the desired statistical moments of the system outputs (such as mean and standard deviation) can be rapidly obtained. The PCE method offers the potential to be significantly more efficient than sampling based UQ methods such as MC simulation, and can show exponential convergence of the error in estimating the mean and standard deviation. Furthermore the method can be applied in a non-intrusive manner i.e. does not require an analytical expression and can work with black-box models such as CAD or FE simulations used in CAT models.

PCE is systematically discussed in detail in [12] where the feasibility of PCE based UQ in tolerance analysis and synthesis is established with:

- A theoretical analysis of the PCE method identifying working principles, implementation requirements, advantages and limitations.
- Establishing of recommendations for PCE implementation in tolerance analysis including methods of PCE coefficient calculation and approach to error estimation.

PCE is subsequently implemented into a PIDO tolerance synthesis framework. The following is a case study which demonstrates the application of the PIDO based approach and highlights the computational cost benefits possible with PCE.

3 Case study - Problem definition

The case study presents a tolerance synthesis problem in which product functionality is defined by both external and internal loading (friction and multi-body dynamics). The case study is a significant extension of the tolerance analysis problem presented in [13].

The product under analysis is a rotary switch assembly used in automotive applications. Positional restraint and a desired resistive switch actuation torque are provided by a spring loaded radial detent acting on the perimeter of the switch body (Figure 2). The cylindrical detent is located in a positioning sleeve within which a helical compression spring biases the cylindrical detent against the switch detent ramp faces. The peak resistive torque is of particular relevance to functionality of the assembly (denoted as a Key Product Characteristics (KPCs)) and depends on: part geometry, part acceleration, spring force, contact forces between components, and friction coefficient between components in contact.

A sufficient resistive torque is required to provide ergonomically and functionally adequate positional restraint with a positive impression of product quality. Excessive variation in the peak resistive torque of manufactured switch assemblies has a negative impact on perceived product quality. The 8 design variables considered in the simulation are shown in Figure 2 and Table 2. The product requirements define a series of constraints and objectives:

- A nominal peak resistive torque of 75 Nmm has been experimentally identified as desirable for the intended application with specification limit set at 75 ± 7 Nmm. This value corresponds to unity value for the C_{pm} Process Capability Index (PCI) as defined in equation (1) [24]. The index increases with any deviation away from a nominal value and is a convenient metric of quantifying manufacturing variation.

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - \tau)^2}} \qquad (1)$$

Where:
τ *: target nominal value*
σ*: standard deviation*
USL: Upper Specification Limit
LSL: Lower Specification Limit

- The rotary switch and positioning sleeve are injection moulded polymer components. The radially acting cylindrical detent is machined mild steel. The steel spring is manufactured on dedicated wire coiling machinery and the associated spring specification limits are based on estimates presented in [13].
- The different materials and manufacturing processes result in specific cost-tolerance characteristics for the associated part parameters. Increased precision typically increases manufacturing costs due to demands for higher precision machinery, increased number of manufacturing steps, and stricter process control [25]. Various cost-tolerance functions have been proposed for representing the manufacturing cost to tolerance relationship for a range of manufacturing processes [26, 27]. The exponential function (Equation (2)) has particular applicability to a range of scenarios and is used in this work [28, 29]. The exponential function represents the tolerance cost, $g(T)$, for a specific tolerance T, as:

$$g(T) = Ze^{-\phi(T-T_0)} + g_0 \qquad \text{Where } T_{min} \leq T \leq T_{max} \tag{2}$$

- *T_{min} and T_{max} define an economically feasible tolerance range.*
- *g_0 and T_0 define minimum threshold cost and tolerances, respectively.*
- *Z and ϕ are curve fitting parameters derived from experimental data.*
- *Costs are specified in cost units.*

- Cost-tolerance curves for each part parameter were specified in consultation with an industrial partner and are presented in Figure 3. The estimated curves indicate the expected cost penalty in terms of the standard deviation of the associated part parameter.

The case study objective is to specify optimal process capabilities for part parameters, such that the following objectives and constraints are addressed (Table 1).

Table 1. Case study objectives and constraints

Objective	Description	Constraint
Maximize C_{pm} of assembly KPC	Maximize the number of assemblies conforming to the peak resistive torque (KPC) specification requirements	The minimum required assembly yield is 99.7% (C_{pm} = 1)
Minimize total tolerance cost	Minimise the total cost of required part tolerances dictated by part parameter specific cost-tolerance curves (Figure 3)	Maximum allowable tolerance cost is 515 cost units as shown for reference in Table 2 and Figure 5. Objectives space of tolerance synthesis for case study.. This value corresponds to the best design identified in prior work with manual tolerance allocation [13].

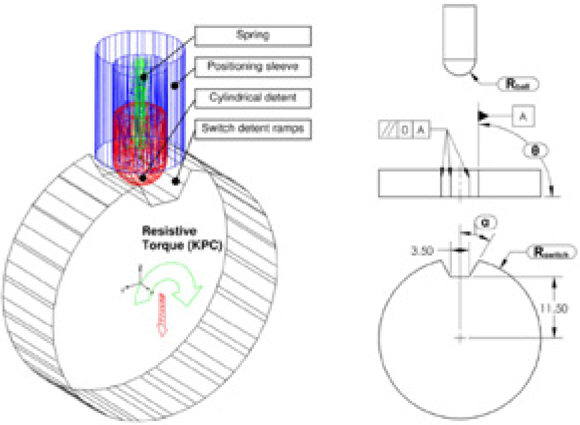

Figure 2. Rotary switch and spring loaded radial detent assembly model
(Note: Linear dimensions in mm. Variation in non-enclosed dimensions not considered in simulation)

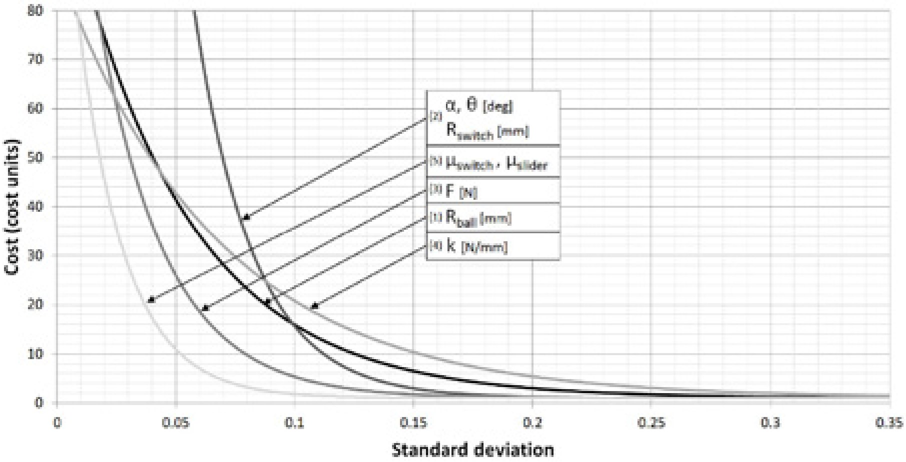

Figure 3. Cost-tolerance curves for part parameters of radial detent assembly

3.1 Simulation model and optimization

A parametric numerical model of the switch assembly was constructed using MSC ADAMS Multi-Body Dynamics modelling software (Figure 2) accommodating the possible variation within geometric and physical parameters (such as spring pre-load, spring stiffness and friction coefficients).

Two PIDO tools were interfaced with MSC ADAMS software according to the PIDO tolerance synthesis platform developed by [12]. UQ was conducted using DAKOTA [30] with process scheduling and optimization carried out using ESTECO modeFRONTIER (Figure 4). All part parameter distributions were assumed to be Gaussian. The tolerance synthesis process consisted of the following stages:

1. Trial standard deviations (i.e. tolerances) for stochastic dimensional, spring and friction parameters of models were selected.
2. Total assembly tolerance cost was calculated and checked for feasibility against established cost constraints.
3. Trial standard deviation were conducted with sparse grid based PCE simulation. For each UQ sampling point the CAE simulation consisted of:
   a. A rotational velocity of 30 degrees per second was imposed on the rotary switch and the interaction of components simulated for 500 ms.
   b. Peak and transient resistive torque were recorded.
4. Moment estimates for the peak resistive torque KPC were returned from the UQ tool and C_{pm} was calculated.
5. Objective function fitness was assessed with a Multi-Objective Genetic optimization Algorithm (MOGA) with emphasised multi-search elitism which shows good performance in preserving high fitness solutions without compromising a global search by convergence to local optima [31]. Following fitness evaluation a new trial standard deviations were generated (return to step 1).

The optimization was terminated at the iteration limit of 300 designs.

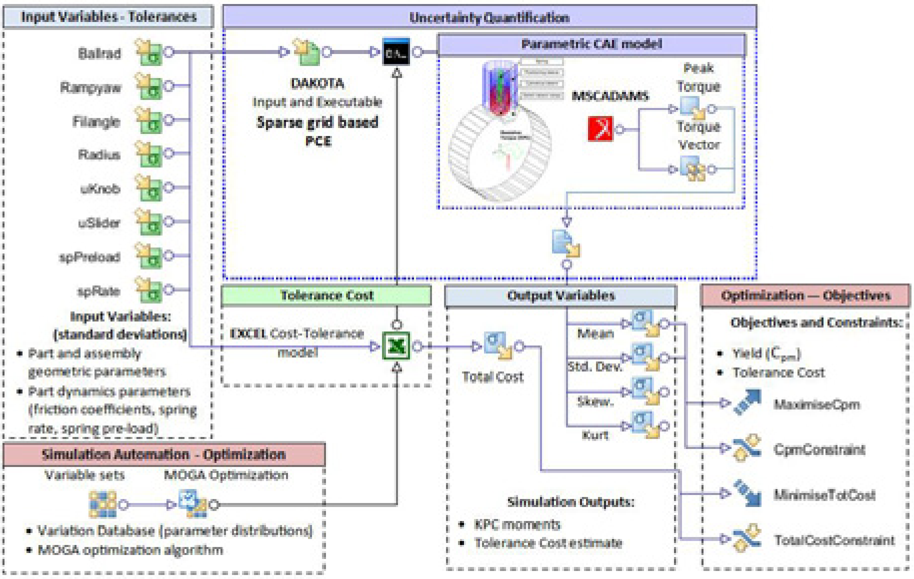

Figure 4. PIDO tolerance synthesis workflow for case study.

3.2 UQ strategy

Sparse Grid (SG) based PCE was incorporated into the PIDO workflow UQ in the switch assembly CAE model [12]. The PCE based mean and standard deviation estimates required 17 model evaluations in total. These were compared against a reference MC estimate of 5000 samples. The differences between the PCE and MC estimates of mean and standard deviation are approximately 2% and 4%, respectively, showing comparable results. The PCE method of UQ drastically reduced the number of model evaluations required for estimation of the mean and standard deviation estimates of the assembly KPCs.

3.3 Optimization strategy

The optimization algorithm employed was a MOGA with emphasised multi-search elitism [31]. The number of generations was 15 with an initial population size of 20. The initial population is a uniformly distributed subset created by applying filtering to a MC sampled base population of 800 designs. The filtering maximizes the separation distance between base points resulting in more uniform sampling of the parameter space [32].

3.4 Simulation results and outcomes

Simulation results are shown in Table 2 and Figure 5. The tolerance synthesis platform was able to identify a design (design #540) with significantly superior performance to the previous best. Compared to prior designs, design #540 achieves a cost reduction of 40% and an increase in C_{pm} of 59%. Each evaluation of the CAE model took 8 seconds on a quad core 3.2 GHz CPU. Process integration overheads amount to a time of approximately 10 seconds per design. The total number of model evaluations was 5100 (17 for UQ and 300 for optimization) resulting in a total simulation time of approximately 14.2 hours. If UQ was conducted through traditional sampling methods such as MC with a relatively small sample size of 1000, the total simulation time would be a comparatively impractical 35 days. With additional time resources the selected design could be subjected to a local refinement to explore the objectives space within the vicinity of the selected optimum design with greater resolution.

The final design (Design ID#540) was validated against a MC reference estimate of 5000 samples. The differences between the PCE and MC estimates of mean and standard deviation were approximately $\delta_\mu = 1\%$ and $\delta_\sigma = 5\%$, respectively, showing negligible differences in estimates.

Table 2 - Case study assembly parameters, associated variation and tolerance synthesis outcomes

Component	***Switch***			***Spring***		***Cylindrical detent***			***Assembly***	
Parameter	**R_{switch}** [mm]	**A** [deg]	**Θ** [deg]	**F** [N]	**K** [N/mm]	**R_{ball}** [mm]	**μ_{switch}**	**μ_{slider}**	***Torque [Nmm]***	***Total Cost** [Cost units]*
Description	*Switch radius*	*Angle of ramp face*	*Yaw angle of ramp face*	*Spring preload*	*Spring rate*	*Ball radius*	*Switch-detent dynamic friction coefficient*	*Slider-detent dynamic friction coefficient*	*Nominal peak resistive torque (KPC)*	*Total Tolerance cost of assembly*
Nominal	15	30°	0°	2	0.400	3	0.150	0.150	75	
Specification Limits +/-	0.250	5°	3°	0.200	0.040	0.190	0.020	0.020	7	
Min.	14.750	25°	-3°	1.800	0.36	2.810	0.130	0.123	68	
Max.	15.250	35°	3°	2.200	0.440	3.190	0.173	0.173	72	
Initial										
μ	15	30°	0°	2	0.400	3	0.150	0.150	75.488	
σ	0.083	1.667	1	0.067	0.013	0.063	0.008	0.008	3.756	
C_{pm}	1	1	1	1	1	1	1	1	**0.620**	
Tolerance Cost	73.104	1	1	13.499	70.536	98.381	112.264	112.264		**482.047**
Manual allocation										
μ	15	30°	0°	2	0.400	3	0.150	0.150	75.404	
σ	0.083	0.833	1	0.033	0.013	0.032	0.008	0.008	1.985	
C_{pm}	1	2	1	2	1	2	1	1	**1.150**	
Tolerance Cost	77.818	1	1	41.137	70.536	98.381	112.264	112.264		**514.399**
Optimised (Design ID #540)										
μ	15	30°	0°	2	0.400	3	0.150	0.150	75.178	
σ	0.145	0.169	2.767	0.027	0.008	0.023	0.019	0.038	1.261	
C_{pm}	0.573	9.890	0.361	2.449	1.671	2.784	0.370	0.222	**1.832**	
Tolerance Cost	79.201	4.228	1	50.655	76.355	9.244	62.771	25.721		**309.175**

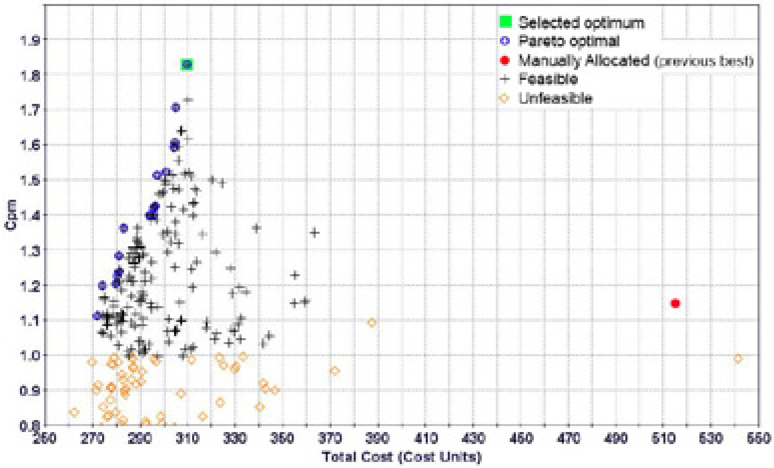

Figure 5. Objectives space of tolerance synthesis for case study.

The computational time could further be reduced by conducting a sensitivity study to assess if the influence of any part parameters on the assembly KPC is negligible. Parameters with low influence could be held fixed during UQ to reduce the number of required integration points. However this was deemed unnecessary, as the objective of this work is to demonstrate the significant computational cost reduction in tolerance analysis and synthesis through application of PCE even in the case of high dimensional problems.

4 Conclusion

Tolerance synthesis in complex product assemblies can be associated with an impractically high computational cost, especially when assessing assembly functionality requires modelling of the effects of loading. The high computation cost is attributable to the traditional use of Monte Carlo (MC) simulation, which although robust, requires a large number of model evaluations for Uncertainty Quantification (UQ).

This work presented a tolerance synthesis case study of an automotive switch assembly under loading, in which a newly developed Process Integration and Design Optimisation (PIDO) approach is applied which addresses the high computational cost of UQ with Polynomial Chaos Expansion (PCE). Optimal tolerances were identified which satisfied desired yield and tolerance cost objectives, while the application of PCE drastically reduced the number of model evaluations required for estimation of the mean and standard deviation estimates of the switch assembly KPCs.

The approach demonstrated in this work has sufficiently high computational efficiency to allow application by practicing engineers for tolerance synthesis to effectively increase product robustness.

References

[1] Spotts, M.F., T.E. Shoup, and L.E. Hornberger, *Design of machine elements*2004: Pearson Prentice Hall.

[2] Merkley, K.G., *Tolerance analysis of compliant assemblies*, 1998, Brigham Young University.

[3] Bergman, B., et al., *Robust Design Methodology for Reliability: Exploring the Effects of Variation and Uncertainty*2009: John Wiley & Sons.

[4] Ebro, M., T.J. Howard, and J.J. Rasmussen. *The foundation for robust design: enabling robustness through kinematic design and design clarity*. 2012.

[5] Ullman, D.G., *The mechanical design process*2003: McGraw-Hill.

[6] Chase, K.W., and Greenwood, W. H., *Design Issues in Mechanical Tolerance Analysis.* ASME Manufacturing Review, 1988. **1**(1): p. 50-59.

[7] Lee, S. and W. Chen, *A comparative study of uncertainty propagation methods for black-box-type problems.* Structural and Multidisciplinary Optimization, 2009. **37**(Copyright 2009, The Institution of Engineering and Technology): p. 239-53.

[8] Liu, S.C. and S.J. Hu, *Variation simulation for deformable sheet metal assemblies using finite element methods.* Journal of Manufacturing Science and Engineering, 1997. **119**: p. 368.

[9] Prisco, U. and G. Giorleo, *Overview of current CAT systems.* Integrated Computer-Aided Engineering, 2002. **9**(Copyright 2003, IEE): p. 373-87.

[10] Shah, J.J., et al., *Navigating the tolerance analysis maze.* Computer-Aided Design and Applications, 2007. **4**(Compendex): p. 705-718.

[11] Shen, Z., et al., *A comparative study of tolerance analysis methods.* Journal of Computing and Information Science in Engineering, 2005. **5**: p. 247.

[12] Mazur, M., *Tolerance analysis and synthesis of assemblies subject to loading with process integration and design optimization tools*, in *School of Aerospace, Mechanical and Manufacturing Engineering*2013, RMIT University: Melbourne, Australia.

[13] Mazur, M., M. Leary, and A. Subic, *Computer Aided Tolerancing (CAT) platform for the design of assemblies under external and internal forces.* Computer-Aided Design, 2011. **43**(6): p. 707-719.

[14] Flager, F., et al., *Multidisciplinary process integration and design optimization of a classroom building.* Electronic Journal of Information Technology in Construction, 2009. **14**(Copyright 2010, The Institution of Engineering and Technology): p. 595-612.

[15] Hiriyannaiah, S. and G.M. Mocko, *Information Management Capabilities of MDO Frameworks.* ASME Conference Proceedings, 2008. **2008**(43277): p. 635-645.

[16] Sobieszczanski-Sobieski, J. and R.T. Haftka, *Multidisciplinary aerospace design optimization: survey of recent developments.* Structural and Multidisciplinary Optimization, 1997. **14**(1): p. 1-23.

[17] Hong, Y.S. and T.C. Chang, *A comprehensive review of tolerancing research.* International Journal of Production Research, 2002. **40**(Copyright 2002, IEE): p. 2425-59.

[18] Nigam, S.D. and J.U. Turner, *Review of statistical approaches to tolerance analysis.* CAD Computer Aided Design, 1995. **27**(Compendex): p. 6-15.

[19] Xiu, D. and G. Karniadakis, *The Wiener-Askey polynomial chaos for stochastic differential equations.* SIAM Journal on Scientific Computing, 2003. **24**(2): p. 619-644.

[20] Ghanem, R.G. and P.D. Spanos, *Stochastic finite elements: a spectral approach*2003: Dover Publications.

[21] Weiner, N., *The homogeneous chaos.* Amer. J. Math, 1938. **60**(4): p. 897--936.

[22] Schoutens, W., *Stochastic processes and orthogonal polynomials*2000: Springer.

[23] Eldred, M. and J. Burkardt, *Comparison of non-intrusive polynomial chaos and stochastic collocation methods for uncertainty quantification*, 2009, American Institute of Aeronautics and Astronautics, 1801 Alexander Bell Dr., Suite 500 Reston VA 20191-4344 USA.

[24] Chan, L., S. Cheng, and F. Spiring, *A New Measure of Process Capability: Cpm.* Journal of Quality Technology 1988. **20**(3): p. 162-75.

[25] Feigenbaum, A.V., *Total Quality Control, 4th Ed.: Achieving Productivity, Market Penetration, and Advantage in the Global Economy*2012: McGraw-Hill.

[26] Wu, Z., W.H. Eimaraghy, and H.A. Eimaraghy, *Evaluation of cost-tolerance algorithms for design tolerance analysis and synthesis.* Manufacturing review, 1988. **1**(Compendex): p. 168-179.

[27] Dong, Z., W. Hu, and D. Xue, *New production cost-tolerance models for tolerance synthesis.* . Journal of Engineering for Industry 1994(116): p. 199-205.

[28] Chase, K.W., et al., *Least cost tolerance allocation for mechanical assemblies with automated process selection.* Manufacturing review, 1990. **3**(Compendex): p. 49-59.

[29] Dong, Z. and A. Soom, *Automatic optimal tolerance design for related dimension chains.* Manufacturing review, 1990. **3**(Copyright 1991, IEE): p. 262-71.

[30] Adams, B.M., Bohnhoff, W.J., Dalbey, K.R., Eddy, J.P., Eldred, M.S., Gay, D.M., Haskell, K., Hough, P.D., and Swiler, L.P.,, *DAKOTA, A Multilevel Parallel Object-Oriented Framework for Design Optimization, Parameter Estimation, Uncertainty Quantification, and Sensitivity Analysis*, 2011, Sandia Technical Report SAND2010-2183, December 2009. Updated December 2010 (Version 5.1) Updated November 2011 (Version 5.2).

[31] Deb, K., et al., *A fast and elitist multiobjective genetic algorithm: NSGA-II.* Evolutionary Computation, IEEE Transactions on, 2002. **6**(2): p. 182-197.

[32] ESTECO, *modeFRONTIER 4 User Manual*2011.