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TOLERANCE DESIGN OF THE JOURNAL-BEARING KINEMATIC JOINT. NUMERICAL ANALYSIS OF THE EFFECT OF THE SHAPE VARIATIONS ON THE PERFORMANCES

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1. Introduction

Macro geometric variations of mechanical components have an important impact on the product performances, as they can compromise its functionality. The knowledge of the link between shape variation and functionality can enable designers to get an optimum choice of tolerance value. If this link is unknown, tolerances are, in general, specified at the higher level of precision that the technology allows or following empirical rules.
In this work the authors deal with the problem regarding tolerance design of the journal – bearing kinematic joint, in which hydrodynamic lubrication conditions are realised. In particular, they propose a simplified numerical approach to analyse consequences of macro geometric variations on the performances of this kind of bearings. While the effects of micro geometric variations on the performances have been studied by many authors (as example, [Patir 1979] and [Venner 1999]), the effects of the shape variations are not so known. In fact just [Hargreaves 1991] and [Srinivasan 1992] study this problem using Montecarlo method, stochastic processes or fractal-based geometry to model shape variation.
The final objective is numerical formulation of new abacuses to be used in the design of such kinematic joints that show how the kinematic joint performances change when the functional feature has not the ideal shape. Of course, the designer can use the abacus starting from a performance objective and evaluating the maximum allowable macro geometric error (*i.e.* the tolerance value) that satisfies the functional requirements.

2. The journal – bearing kinematic joint

A journal – bearing kinematic joint is schematically represented in fig.1. The journal, *p*, is usually the rotating element of the joint and it is part of a shaft (*i.e.* the rotor of a machine). The bearing, *c*, operates like a constraint as it forces the shaft to rotate around a fixed axis. It is fixed to the support structure, *s*, of the rotor. The journal transmits a radial force, constituting the bearing load. In order to realise a radial clearance, the journal diameter has to be smaller than the bearing one.
When the shaft doesn't run (its angular velocity is zero) and the radial force direction is down, due to the radial clearance, journal and bearing come in contact along a generatrix.
When the shaft rotates the lubricant comes between journal and bearing and generates a pressurised layer. Under stationary conditions the shaft axis and the bearing axis will not coincide; their distance is named journal eccentricity, *e*. In such conditions the shaft axis rotates around the bearing axis creating a convergent – divergent meatus.

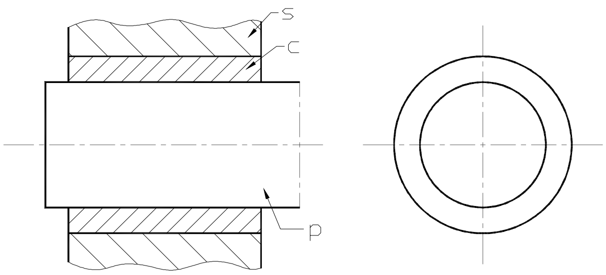


Figure 1. Journal – Bearing kinematic joint

In this way the pressure field generated within the meatus supports the load acting on the bearing. The use of such bearings is recommended when high loads and high angular velocities are faced.

3. Numerical analysis of the journal cylindrical bearings

The study of the journal cylindrical bearings (and generally of all the mechanical parts in which hydrodynamic lubrication conditions are realised) is faced through the solution of the *Reynolds equation* (also known as lubrication equation), which allows to obtain the pressure field distribution within the meatus when specific hypotheses on the lubricant characteristics and on its motion within the meatus are satisfied [D'Agostino 1992]. Once this pressure field distribution is known it is possible to calculate the load carrying capacity for the bearing design.
In fig. 2 it is schematically represented the geometry of the system on which the Reynolds equation in its two-dimensional form (1) is applied.

$$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(h^3 \frac{\partial p}{\partial z}\right)=6\mu U \frac{\partial h}{\partial x} \tag{1}$$

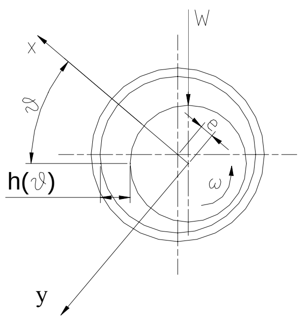


Figure 2. Reynolds equation parameters in the journal – bearing kinematic joint

In the table 1 the Reynolds equation parameters are briefly explained.

Table 1. Reynolds equation parameters

B	Axial dimension of the bearing	θ	Circumferential direction
R	Radius of the bearing, R = D/2	U	Journal surface velocity
r	Radius of the journal	p	Pressure
c	Radial clearance, c = R - r	μ	Viscosity
e	Eccentricity	h	Film thickness, $h(\theta) = c(1+n\cos\theta)$
n	Eccentricity ratio, n = e/c	h_{min}	Minimum thickness
W	Extern load transmitted by the journal	φ	Attitude angle

3.1 Numerical Integration of the Reynolds equation

In literature several authors [D'Agostino 1992], regarding the integration of the Reynolds equation, make reference to the theory developed by *Sommerfeld* which provides the results following an analytical approach but introducing a simplifying hypothesis (infinite bearing): $B >> R$. Other hypothesis (short bearing) was developed to obtain the analytical solution but none of them can properly interpret the real cases. Thus it is necessary to model "the finite bearing" which can't be solved analytically.

3.1.1 Finite difference method

Following the numerical approach [Pinkus 1961], [Gutes 1997] it is possible to solve the steady – state Reynolds equation for hydrodynamic journal bearing and to determine the pressure field in the fluid film. This approach foresees five steps.

i) Non – dimensional equation (1) with the following parameters:
$x = r\theta\ (0 \le \theta \le 2\pi)$; $z = BZ\ (0 \le Z \le 1)$; $h = cH$; $p^* = 6\mu Ur/c^2$; $p = \gamma\, p^*$; $k = r/B$.
Equation (2) is obtained:

$$\frac{\partial H^3}{\partial \vartheta}\frac{\partial \gamma}{\partial \vartheta} + H^3\frac{\partial^2 \gamma}{\partial \vartheta} + k^2\left(\frac{\partial H^3}{\partial Z}\frac{\partial \gamma}{\partial Z} + H^3\frac{\partial^2 \gamma}{\partial Z^2}\right) = \frac{\partial H}{\partial \vartheta} \tag{2}$$

ii) The inner bearing surface is developed in a plane and a mesh is built (fig. 3).

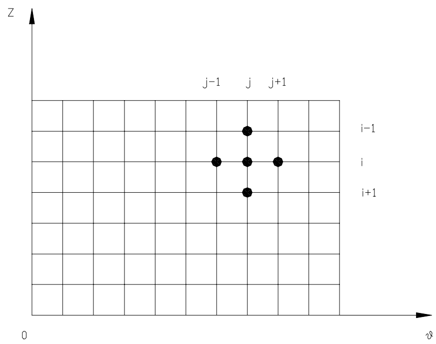


Figure 3. Finite difference meshFigure

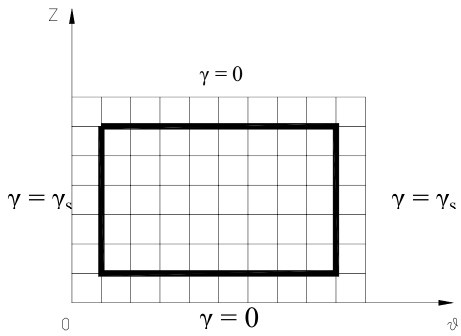


Figure 4. Boundary Conditions

iii) In the generic (i, j) point of the mesh, the derivatives of the $\gamma(\theta, Z)$ function (representing the non – dimensional pressure) and the $H(\theta, Z)$ function (representing the non – dimensional thickness of the film) are approximated through their incremental factors.

iv) Boundary conditions, as shown in fig. 4, are defined (γ_s is the supply pressure; $\gamma_0 = 0$ is the atmospheric pressure).

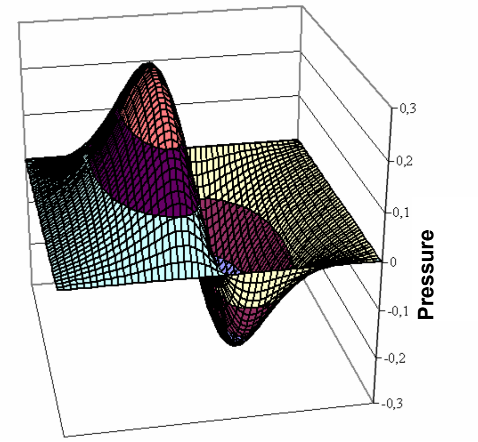
v) The last step foresees the solving of a linear system of equations, **[A] [X] = [B],** where: **[A]** is the coefficients matrix, **[X]** is the unknown pressures vector, and **[B]** is known terms vector.

The solution of this system provides the pressure's field in the fluid film. The more the mesh is dense, the more the solution will be exact.

The solution points out that while in the meatus convergent region the pressure is positive, in the divergent region it is negative (fig. 5). In order to obtain the load carrying capacity only the positive region of pressure field is integrated according with the "*half Sommerfeld condition"* [D'Agostino 1992] which assumes pressure to be zero in the divergent region (fig. 6).

3.2 Bearing performances in the ideal case

The authors have developed an algorithm (named "SMOOTH") written in Fortran language in order to solve the Reynolds equation and to build the project curves related to journal bearing. The program has been validated under the hypothesis of ideal surfaces between the two pieces of the kinematic joint. To validate the programme the extracted curves have been compared with those supplied by manuals [D'Agostino 1992], [Jacazio 1992].



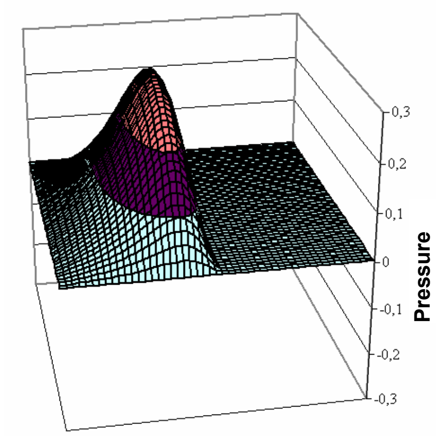


Figure 5. Pressure's field: "*Full Sommerfeld condition*" **Figure 6. "*Half Sommerfeld condition*"**

For examples of these first outputs of the programme in fig. 7 the project curves regarding the minimum thickness of the film (h_{min}) in function of the Sommerfeld number ($1/\Delta = \mu Bur^2/\pi Wc^2$) for several values of the ratio B/D are shown. In fig. 8 the project curves regarding the circumferential oil flow in function of the Sommerfeld number for several values of the ratio B/D are shown.

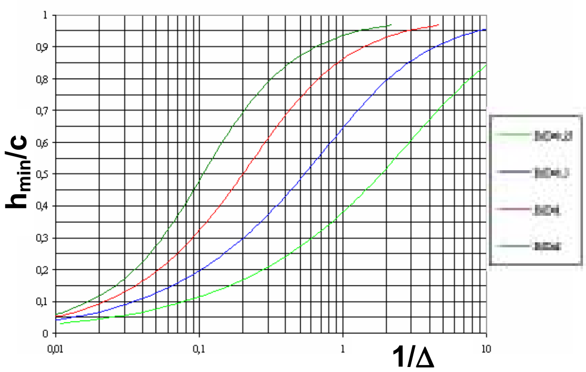


Figure 7. Minimum thickness of the film

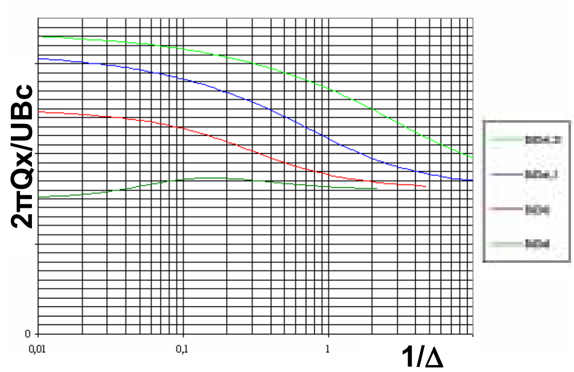


Figure 8. Circumferential oil flow

4. The simulation of the shape variation.

While the effects of micro geometric variations on the performances have been studied by many authors [Patir 1979], [Venner 1999], the effects of the shape variations are not so known. Nowadays often the designers assign a very restricted tolerance value to hide their ignorance regarding the influence of the shape variation on the bearing's functional parameters. So it should be useful to quantify the difference between the behaviours of the real bearings and the project curves extracted under the hypothesis of ideal surfaces.

A shape variation from the ideal surface determines continuous variation of meatus thickness along *Z*. Therefore the meatus thickness can be modelled by a stochastic process and it is a random variable in each section. Having formulated hypothesis on its distribution, the shape variation can be simulated through the Monte Carlo method. As the cylindricity tolerance prescribes that the real inner bearing surface has to be contained within two coaxial cylinders whose distance is the tolerance value, *t* (fig. 9), it is possible to simulate the real shape subjected to this constraint.

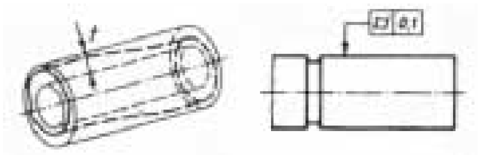


Figure 9. Cylindricity tolerance

4.1 Design of Experiments

As the Montecarlo method provides reliable results only executing many iterations of the procedure, some simplifying hypotheses have to be introduced.

i) Considering the more critical conditions, only points belonging to the two borders of the tolerance range and to the ideal surface are taken into account. So the meatus thickness (h) can assume three values: (h-t/2), (h), (h+t/2).

ii) Considering only macro geometric variation it is possible to take into account a discrete number of surface points to be associated with an error.

iii) Even if the numerical solution of the Reynolds equation has been obtained for its two dimensional form (θ and Z variables), in order to save calculation time, only the θ dimension is taken into account.

iv) Cause to the "Half Sommerfeld condition" the range between π and 2π, in which the meatus is divergent, is meaningless.

v) In order to study macro geometric shape variation along θ direction, the authors have fixed 5 "*stations*" in the range $[0, \pi]$: 0, $\pi/4$, $\pi/2$, $3\pi/4$, π. In every station is simulated the shape variation, which can assume one of the three values (h-t/2), (h), (h+t/2).

vi) In order to even simplify the calculations an open traverse has been considered (in this first work the description of a general method to relate the shape variation to the bearing's behaviours was considered more important than a hard code). In fig.10 the variation of the thickness of the meatus, when a shape variation occurs, is shown.

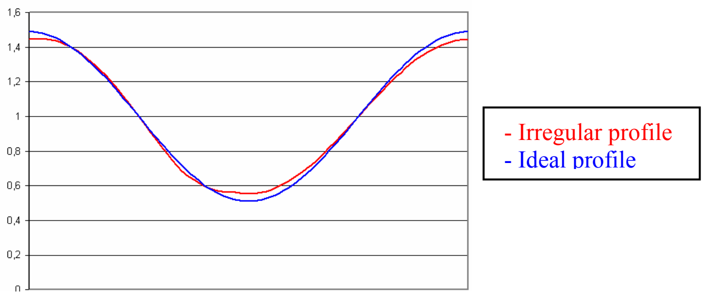


Figure 10. Variation of the thickness of the meatus

Even though these simplifying hypotheses, the estimation of the computation time needed to run the full factorial design with 3^5=243 possible configuration, suggests to use the fractional factorial design [Box *et al.* 1978], [Park 1996], [Montgomery 1996]. To this end the $L_{27}(3^{13})$ orthogonal array has been selected. So the computer experiment phase has been carried out only through 27 simulation (that take about 48 hours of runtime on a pc platform).

4.2 Carrying out the experiments

Starting from the SMOOTH algorithm, a new one, FACET, has been developed taking into account a non-dimensional tolerance parameter and fixing the B/D ratio to 1. Since in the algorithm the input variable strongly connected with the bearing surface is the radial clearance, c, it can be defined again as: $c = c^*(1 \pm err/2)$, where c^* is the nominal radial clearance (see table 1) and $err = t/c^*$ is the non dimensional tolerance parameter ($0.1 \le err \le 1$). The FACET algorithm simulates the 27 facet surfaces of the fractionated test's planning entering an *err* value. For each *err* value and for each facet surface the programme provides some diagrams relating the bearing's characteristics (minimum thickness h_{min}, circumferential flow Q_x, axial flow Q_z, attitude angle φ, friction coefficient fr, and temperature gradient ΔT) to the Sommerfeld number $1/\Delta$.

4.3 Data Analysis

The fractional factorial design leads to a simply additive model. It hypothesises that the interactions between the control factors are not significant. In order to confirm this working hypothesis the process has been run for several eccentricity values and for two err values: 0.1 and 0.2. Due to the great amount of data obtained the Pareto ANOVA analysis [Park 1996] has been used in order to build for each characteristic the "*contribution ratio*" curve of each station in function of the eccentricity. In this way has been explained the influence that each station have on each characteristic. Confronting the diagrams concerning the two error, *err*, values (*i.e.* the diagrams regarding the Sommerfeld number are shown in fig.11), the following information has been obtained:

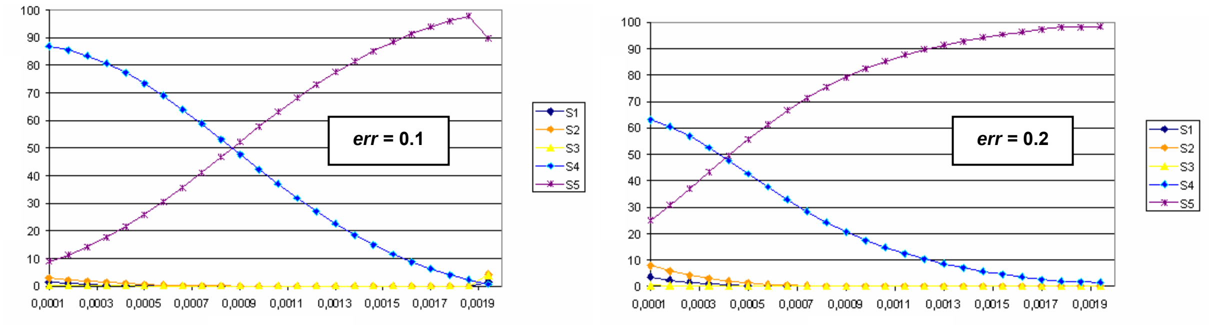


Figure 11. Contribution ratio in function of the eccentricity for the Sommerfeld number

i) The more significant stations are last two (4 and 5) corresponding to the convergent part of the meatus where the pressure gets the maximum value. This is in agreement with the physical model of the journal- bearing joint.
ii) Varying the non-dimensional tolerance value quite coherent results are obtained. In fact they seem to be affected only by the scale factor between the two *err* values.

4.3.1 Determination of the "best profile" and the "worst profile"

The estimation of the medium effects pointed out by the Pareto ANOVA analysis allows determining the factor's combination that make the analysed performance minimum or maximum. Thus for each program output there are some level combinations in the various stations that optimise, for example, the Sommerfeld number rather than the circumferential flow. Facing the study from an engineering point of view the worst conditions have to be checked and then affecting them with a security factor. So it is important to give more importance to the "worst profiles". In particular it is checked the combination that minimises the Sommerfeld number; further it is checked the combination that maximises:

- the friction coefficient;
- the temperature gradient;
- the circumferential oil flow;
- the axial oil flow.

The study has pointed out that five profiles are sufficient to get, for each simulated eccentricity, the maximum and minimum value of all the significant behaviours.

4.3.2 Additive model validation

In order to validate the additive model, a simulation generating these 5 profiles has been arranged. As the curves generated by this simulation are extern to those one generated using the fractionated test's planning and the same *err* value, the design parameters are not interacting and the additive model is applicable (fig. 12).

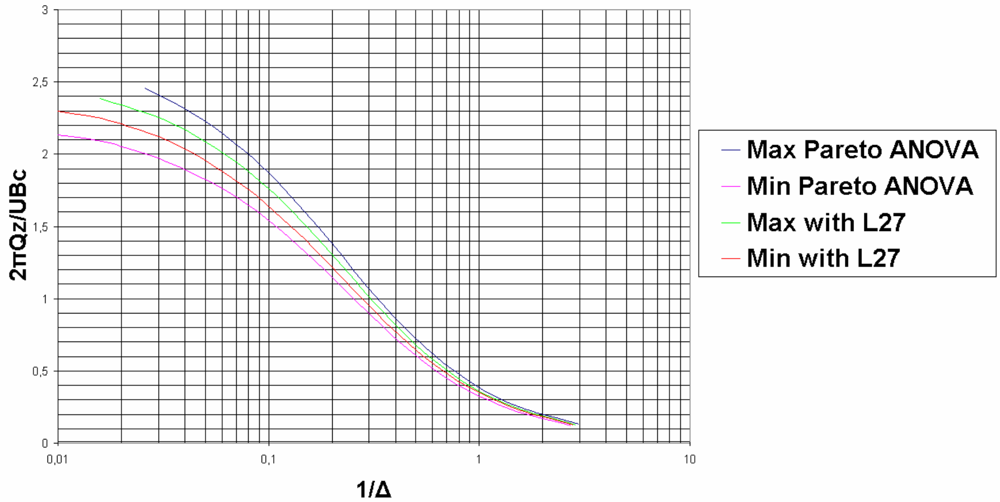


Figure 12. Additive model validation: Axial oil flow as function of the Sommerfeld number

4.4 Kinematics joint performances for several tolerance values

Since the model has been validated, in order to obtain the extreme curves of the variability range for each performance, several simulations have been made varying the *err* value from 0.1 to 1. The simulations pointed out a light influence of the shape variation on the Sommerfeld number value, the friction coefficient, the temperature gradient and the axial oil flow, as well as a considerable influence on the circumferential flow (fig. 13).

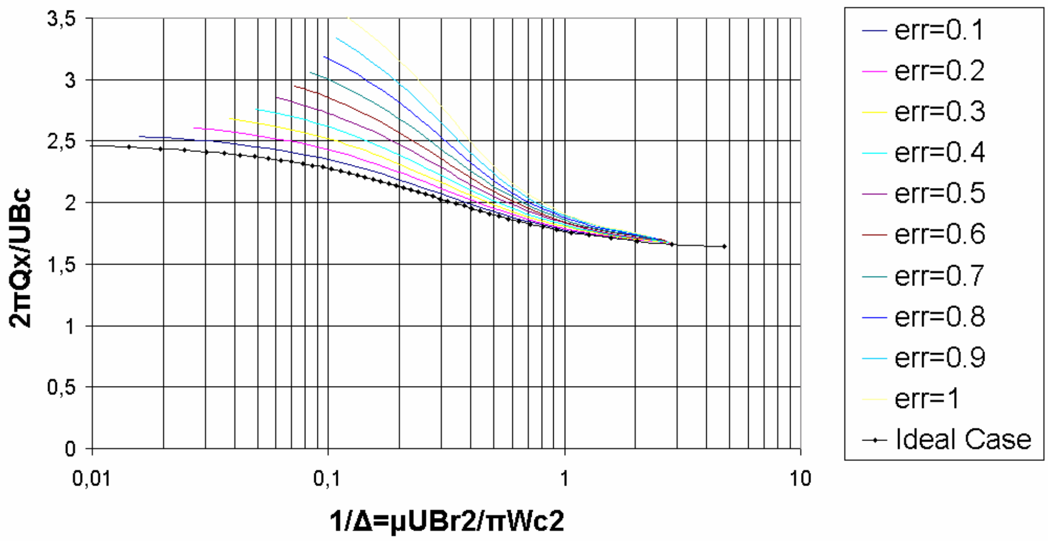


Figure 13. Circumferential oil flow as function of the Sommerfeld number for several tolerance values

4.5 Tolerance abacuses

In order to give a practical tool to designer some tolerance abacuses are proposed. These show, for a fixed value of the bearing load carrying capacity, the level of each performance as function of *err* value, both for the best profile and for the worst profile. The more consistent results have been obtained from the abacus related to the circumferential oil flow. The best profile influences this characteristic till the 2.32%; the worst profile (which maximises the performance) influences the characteristic even till the 32.2% (fig. 14).

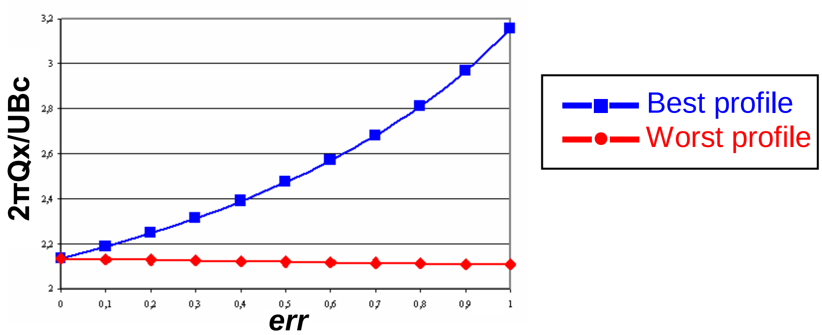


Figure 14. Tolerance Abacus: circumferential flow as function of *err* for a fixed Sommerfeld number 1/Δ = 0.2

5. Conclusions and future works

In conclusion in this work the first results of a method to evaluate shape variation influence on the performances of the journal – bearing kinematic joint are shown. For the first time a design of experiments methodology is proposed to simulate the shape variation to save computational time compared to an extensive application of the Montecarlo method. Furthermore the authors have proposed some abacuses to study the performances of the kinematic joint as function of the tolerance. These abacuses can be used directly by the designer. These curves allow the functional choice of geometrical tolerances.
This work is the first step of a research project about tolerance design through modelling the shape variation effects on functional requirements of kinematic joints. To this end, the algorithm realised by the authors has been arranged in order to allow future shape variation modelling removing some simplified hypotheses just adopted.

Acknowledgement

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References

Box, G.E.P., Hunter, J.S., Hunter, W.G., "Statistics for Experimenters", Wiley, New York, 1978.
D'Agostino, V., "Fondamenti di Tribologia", Cuen Napoli, 1992.
Gutes, M, Castilla, R., Codina, E., "Resolución numerica de la Ecuación de Reynolds para un cojinete cilindrico", Proceedings of XIII Congreso Iberoamericano de Ingeniería Mecánica, La Habana, Cuba, 1997.
Hargreaves, D.J., "Surface waviness effects on the load-carrying capacity of rectangular slider bearings", Wear, No.145, 1991, pp 137-151.
Jacazio G., Piombo B., "Meccanica applicata alle macchine", Vol. II La trasmissione del moto, Levrotto&Bella, 1992.
Montgomery, D. C., "Introduction to statistical quality control", John Wiley & Sons, 1996.
Park, S. H., "Robust design and analysis for quality engineering", Chapman & Hall, 1996.
Patir N. , Cheng H.S., "An average flow model for determining effects of three dimensional roughness on partial hydrodynamic lubrication", ASME Journal of lubrication technology, January 1978, vol.100.
Pinkus, O., Sternlicht, B.,"Theory of Hydrodynamic Lubrication", McGraw-Hill, Inc., 1961.
Srinivasan R.S. , Wood K.L. "Fractal-based geometric tolerancing for mechanical design", Design theory and methodology, ASME, De-vol.42, 1992.
Venner C.H. , Lubrecht A.A., "Elastohydrodynamic lubrication of rough surfaces", J07797 ImechE 1999, Proc Instn Mech Engrs, vol. 213 part j.

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