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TOLERANCE ANALYSIS OF MECHANISMS TAKING INTO ACCOUNT JOINTS WITH CLEARANCE AND ELASTIC DEFORMATIONS

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ABSTRACT

Geometrical deviations can influence the functionality of technical systems in motion. Geometrical deviations due to manufacturing discrepancies are limited by tolerances which are defined during the design process. For cost-efficient tolerance allocation, it is important to evaluate the impact of possible geometrical deviations on the functionality of the whole system as soon as possible.

In this paper, an integrated tolerance analysis approach is developed for a crank mechanism which is subject to time-depending operating conditions and whose components have time- and load-depending positions. The analysis results are the crank angle depending variation of the defined functional dimension (piston position) and the influence of each toleranced parameter. The tolerance analysis model takes into account link length deviations, clearance at the conrod big end bearing and elastic deformation of the crankshaft.

Keywords: Tolerance analysis, motion behavior, clearance, elastic deformation, crank mechanism

1 INTRODUCTION

The success of product development and production processes depends largely upon ensuring the functional capabilities of a product and, at the same time, taking into account the production costs at an early stage of the product life-cycle. The functionality of technical systems in motion is determined by the interaction of their components. This interaction is influenced by small geometrical variations of the components which can originate from manufacturing discrepancies and from deformations of the components. The manufacturing discrepancies are limited by the defined tolerances during the design process. The designer often tends to define too tight tolerances so that the proper functionality of the product is guaranteed. However, it is not possible to persist on this viewpoint due to the fact that the competitive pressure forces the companies to produce their products as cheap as possible. A significant manufacturing cost reduction can be achieved by (re)allocating the tolerance values in the right way.

For an efficient tolerance allocation process, it is necessary to be able to foresee the impact of possible geometrical deviations (at the component level) on the functionality of the assembly. This can be done by (computer-aided) tolerance analysis systems. The problem is, that in most cases only the assembly process is considered and not the whole motion sequence of the mechanism at operating state. In addition to this, acting forces on the system are left out. On the one hand, these forces lead to deformations of the components and on the other hand they influence the relative displacement of the components at a joint with clearance. It is important to consider both effects during the tolerance analysis process to get to a proper tolerance allocation.

In this paper, an integrated tolerance analysis approach shall be presented which can be applied to non-ideal mechanisms and which takes into account manufacturing discrepancies, joints with clearance and deformations. All explanations are exemplified by means of a crank mechanism inside a four stroke, single cylinder combustion engine. At the beginning, the different steps of tolerance analysis in general are briefly described. Existing approaches for the analysis of non-ideal mechanisms shall be classified. After that, the given challenges for mechanisms, and especially the demonstrator crank mechanism, against the background of tolerance analysis are shown in chapter 3. The following chapter then describes the integrated tolerance analysis approach, especially the procedures to integrate deformations of the components and joints with clearance. The calculations which were performed for the crank mechanism and their results are shown in chapter 5. The last chapter summarizes the paper and points out future prospects.

2 EXISTING APPROACHES FOR TOLERANCE ANALYSIS OF MECHANISMS

Generally, the tolerance analysis procedure can be divided into three steps. At first, the mathematical relation between the functional dimensions of the system and the toleranced component parameters has to be established (functional relationship). It is important to realize that especially the common geometrical tolerances are not suitable for mathematical calculations. This leads to the need of an explicit tolerance representation and a suitable transformation procedure. Existing representation schemes are shown in [1] and [2]. The second step analyzes the variation of the functional dimensions for varying component parameters. For this purpose, different tolerance analysis methods can be applied to the functional relationship. Arithmetical methods calculate the variation of the functional dimensions for maximum and minimum values of the toleranced parameters. Statistical methods (as shown in [3]) facilitate the integration of production information about the geometrical properties. The influence of each toleranced parameter on the variation of the functional dimensions can be evaluated by using contributors analysis methods. The last step is the representation and interpretation of the simulation results.

In the following, the focus is on geometrically non-ideal systems in motion. The basic compatibility of multi-body systems and tolerance analysis methods is shown in [4]. The crank's angular speed and driving torque of a crank mechanism in a combustion engine are analyzed depending on dimensional tolerances in [5]. In addition to dimensional variations, the motion behavior of a mechanism can be influenced by joint clearances. This aspect is considered in [6-9]. There are different ways of modeling joints with clearance by inserting a clearance vector into the functional relationship. At first, the clearance vector can be defined according to worst-case [6] and stochastic scenarios [7]. Another possibility is to determine the clearance vector due to a certain joint force [8]. In [9] lubrication effects at joints with clearance are taken into account in the analysis of the mechanical error. Mechanisms which are comprised of higher kinematic pairs in general and especially gears are treated in [10] and [11].

The aspect of an analysis of tolerances and deformations is considered in [12-17]. The focus in [12-15] is on the assembly of compliant parts. A general approach to couple deviation zones (due to elastic deformations) and statistical tolerance zones is shown in [16]. In [17], approaches for the analysis of tolerances and deformations in mechanisms are presented.

It can be observed that all these activities deal with different aspects concerning the setting up of the functional relationship, the use of different tolerance analysis methods and the representation of the results. A holistic approach which integrates different modeling strategies for mechanisms with different kinds of deviation and the possibility to apply different tolerance analysis and result representation methods does not exist.

3 CHALLENGES FOR THE DEMONSTRATOR CRANK MECHANISM

Mechanisms are often characterized by precisely coordinated motions of the components. Geometrical deviations of the components can lead to undesirable collisions. This aspect justifies the need of tolerance analysis. For the demonstrator crank mechanism, collisions of the piston and the valves or the piston and the balance weights of the crankshaft can occur. Consequently, it is important to analyze among others the piston position in relation to the principal axis of the crankshaft as the functional dimension of the mechanism.

The result of tolerance analysis is on the one hand the information about the variation of the functional dimensions for varying toleranced (geometrical) input parameters and on the other hand the information about the contribution of each toleranced parameter to the variation of the functional dimension. The specific problem for all systems in motion with variable positions of the components is that the variation of the functional dimensions and the contribution of each parameter to the variation are changing during the motion sequence. The consequence is a much more complicated situation with regard to the optimization of tolerance allocation.

In figure 1, the already performed analyses for the demonstrator are shown [18]. Deviations in link lengths, position deviation of the cylinder axis and clearance at the conrod big end bearing were considered for the evaluation of the piston position. The results were the crank angle depending variation of the piston position and the crank angle depending contributors situation.

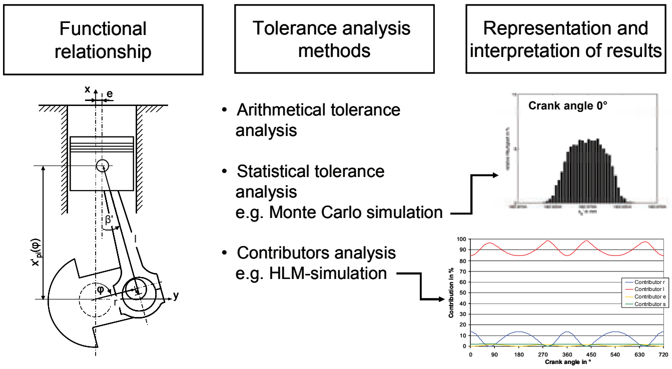


Figure 1. Existing tolerance analysis results for crank mechanism

The clearance at the conrod big end bearing allows a displacement of the conrod relative to the crankshaft which depends on the acting joint force. In [18] the displacement is defined to be in direction of the joint force. In this paper, this model shall be compared with a second model where the hydrodynamic behavior at the joint is taken into account.

Beside the joints with clearance, the functional dimension piston position is affected by elastic deformations. In [18], all the components were considered to be rigid. In the following, the elastic deformation of the crankshaft shall be evaluated. It has to be kept in mind that the elastic deformation depends, as well as the displacement at joints with clearance, on the varying acting forces in the mechanism.

The overall goal is to get an idea of the varying influence of manufacturing discrepancies, joints with clearance and elastic deformations on the functional dimension depending on the mechanism position.

4 INTEGRATED TOLERANCE ANALYSIS APPROACH

In the following, a model shall be set up to evaluate the influence of different kinds of deviation on the functional behavior of the mechanism. As already mentioned, the piston position is declared to be the functional dimension which shall be analyzed. Related to the idea of vectorial tolerancing [19, 20, 21], the kinematical relations, the displacement at the joint with clearance and the deformation are represented by vectors. The functional dimension is the result of a stack-up of the several vectors according to the interaction of the components.

The first step of mechanism analysis is the evaluation of the kinematics. In section 4.1, the kinematical analysis is considered associated with the problem of manufacturing discrepancies. For the integration of joints with clearance and elastic deformations, it is required to have knowledge of the acting forces on the mechanism. Consequently, in section 4.2 the calculation of the necessary forces is briefly described. In the following two sections, it is shown how the geometrical deviations due to joints with clearance and deformations can be taken into account in the tolerance analysis model.

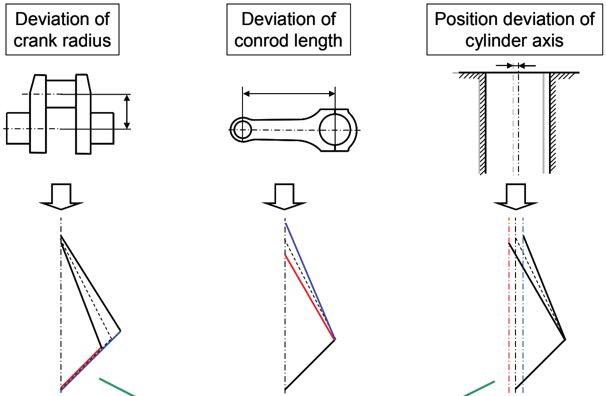
4.1 Kinematics

The kinematical analysis of a mechanism evaluates the motion of the components (position, velocity and acceleration) related to a global coordinate system. The basic relation for the piston position of a centric crank mechanism related to the main axis of the crankshaft without any geometrical deviations is:

$$x_{pi}(\varphi) = r \cdot cos\,\varphi + l \cdot \sqrt{1-(\lambda \cdot sin\,\varphi)^2} \qquad (1)$$

x_{pi}: piston position, φ: crank angle, r: crank radius, l: conrod length, λ=r/l: conrod ratio.

Manufacturing discrepancies can be taken into account in the kinematical analysis. They can change the basic relation for the functional dimension in the way that the existing parameters are set to deviated values or that additional parameters have to be inserted into the functional relationship. Examples for these possibilities are shown in figure 2.



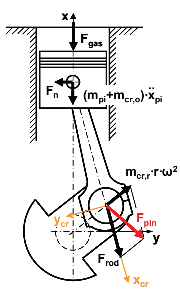
$$x_{pi}(\varphi) = r \cdot \cos\varphi + l \cdot \sqrt{1 - \left(\lambda \cdot \sin\varphi - \frac{e}{l}\right)^2}$$

Figure 2. Impact of different manufacturing discrepancies on functional relationship

The velocity and the acceleration of the piston are the first and the second derivative (with respect to time) of the piston position. The kinematics is afterwards necessary for the calculation of the forces.

4.2 Calculation of forces

For the evaluation of the relative displacement of two components at a journal bearing as well as for the evaluation of the elastic deformations of the components, it is needed to have knowledge of the acting forces. The crank mechanism is affected by the gas force due to the pressure inside the cylinder and the forces due to oscillating and rotating masses. For the analyses in section 4.3 and 4.4, the conrod big end bearing force is needed. The calculation procedure for the joint force F_{pin} which acts from the conrod onto the crankpin is shown in [18]. Figure 3 depicts the relevant forces which influence F_{pin}, relevant calculation parameters and the polar diagram of the joint force F_{pin}.



Calculation parameters	Unit	Value
Crank radius r	mm	45
Conrod length l	mm	138
Piston diameter D_{pi}	mm	84
Piston mass m_{pi}	g	875
Conrod mass m_{cr}	g	805
Maximum cylinder pressure p_{max}	bar	180
Rotational speed n	min^{-1}	3000

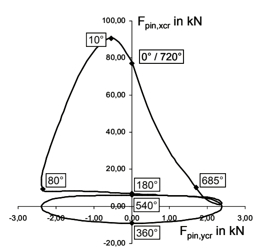


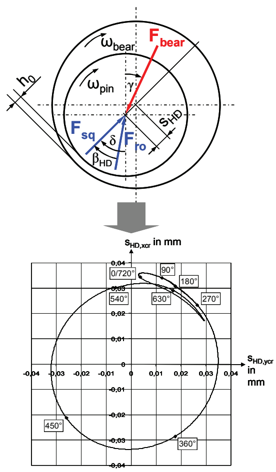
Figure 3. Left: Acting forces on the crank mechanism, Middle: Relevant calculation parameters, Right: Polar diagram of the joint force F_{pin}

The polar diagram of the joint force F_{pin} on the right hand side gives information about the value and the direction of F_{pin} depending on the crank angle. The x_{cr}-y_{cr}-coordinate system is always in line with the conrod during the whole motion of the crank mechanism. One motion cycle means that the crank mechanism passes all four strokes, the crankshaft performs two rotations and the piston moves twice from top dead center to bottom dead center and back. At φ=360° the charge changing takes place.

4.3 Hydrodynamic consideration of a joint with clearance

The bearings inside a combustion engine are subject to time-depending joint forces. Nowadays, the bearings are in most cases journal bearings with hydrodynamic lubrication. The pins move inside the bearing shells in a certain way during one working cycle. The motion path of the bearing pin is defined by the balance of the external joint force and the forces which are generated in the lubricant film (as a result of the pressure build-up due to 1. radial motion and 2. tangential motion of the pin). Normally, this path is the basis for the estimation of the reliability and it is the starting point for the also necessary calculation of friction, oil flow rate and maximum pressure in the lubricant film. Within the scope of tolerance analysis, the motion path shall be used to define the displacement condition of the components at a joint with clearance. This joint condition is necessary for the first step of tolerance analysis which is the definition of the functional relationship.

The calculation of the motion path can be carried out, for example, by the method of superposed hydrodynamic load portions of HOLLAND (for a description of the method in detail see [22]). The external bearing force F_{bear} (which is the reaction force of F_{pin} in section 4.2 and which acts from the pin onto the bearing shell), the hydrodynamic forces F_{ro} (pressure build-up due to pure rotation) and F_{sq} (pressure build-up due to pure squeezing out) are balanced. Using the relations for the similarity coefficients of Sommerfeld So_{ro} and So_{sq} as well as the relation for the relative angular velocity ω_{res}, the differential equations for the time-depending change of the eccentricity ε and the orientation angle of the eccentricity δ can be achieved. The eccentricity ε and the angle δ are the defining parameters for the motion path of the pin. By introducing time steps for the resulting differential equations, the equations for the change of eccentricity $\Delta\varepsilon$ and orientation angle $\Delta\delta$ with a given change in the crank angle $\Delta\varphi$ can be set up. The values for the subsequent calculation step can then be achieved by adding $\Delta\varepsilon$ and $\Delta\delta$ to the actual values. Figure 4 shows the geometrical relationships inside the bearing and the basic equations for the calculation of the motion path. In addition to this, the motion path of the conrod relative to the crankpin (more precisely of the center point of the conrod big end bearing shell to the center point of the crankpin) is displayed in the x_{cr}-y_{cr}-coordinate system.



$$F_{bear} \cdot \cos(\delta - \gamma) = F_{ro} \cdot \cos(\beta_{HD}) + F_{sq}$$

$$F_{bear} \cdot \sin(\delta - \gamma) = F_{ro} \cdot \sin(\beta_{HD})$$

Approximation equations for So_{ro}, So_{sq}, β

$$\Delta\varepsilon = \frac{\Delta\varphi \cdot \pi}{\omega \cdot 180°} \cdot \frac{F \cdot \psi^2}{b \cdot d \cdot \eta \cdot So_{sq}} \cdot \left[\cos(\delta - \gamma) - \left| \frac{\sin(\delta - \gamma)}{\tan \beta_{HD}} \right| \right]$$

$$\Delta\delta = \frac{\Delta\varphi}{\omega} \cdot \left[\frac{\omega_{pin} + \omega_{bear}}{2} - \frac{F \cdot \psi^2}{b \cdot d \cdot \eta \cdot So_{ro}} \cdot \frac{\sin(\delta - \gamma)}{2 \sin \beta_{HD}} \right]$$

Extension to motion path of bearing pin

$$\varepsilon_{i+1} = \varepsilon_i + \Delta\varepsilon \qquad \delta_{i+1} = \delta_i + \Delta\delta$$

with initial values: $\varphi_0 = 0°$, $\varepsilon_0 = 0{,}8$, $\delta_0 = \gamma(\varphi_0) = 180°$

Figure 4. Hydrodynamic consideration of a joint with clearance

As already mentioned in chapter 3, the displacement at the conrod big end bearing which is achieved by the hydrodynamic consideration shall be compared with the displacement which is defined to be in direction of the joint force. In both cases, the clearance can be taken into account in the tolerance analysis procedure by inserting an additional clearance vector into the functional relationship for the piston position. The clearance vectors of both models differ in their lengths and spatial directions. Figure 5 gives an overview over the geometrical relations which are relevant for the transformation of the displacement into the x_{cr}-y_{cr}-coordinate system. It has to be kept in mind, that always the conrod is displaced relative to the crankpin and not vice versa.

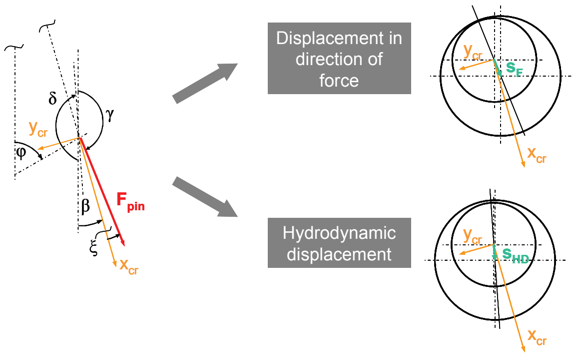


Figure 5. Relevant geometrical relations for coordinate transformation (depicted for φ=60°)

4.4 Analytical calculation of the elastic deformation of the crankshaft

The hydrodynamic consideration of joints with clearance in section 4.3 deals with the connection conditions for the components depending on the acting forces and the physical conditions inside the bearing. The goal of this section is to have a look at the components themselves and their elastic deformations.

In the following, the elastic deformation of the crankshaft is calculated. As a first approach, an analytical model of the deformation is used in this paper. The relevant forces (which lead to bending and torsion) are the joint force F_{pin} and the forces due to the rotating masses of the crankshaft (see figure 6). Equivalent masses represent the rotating masses of the crankshaft throw and the balance weights [23, 24].

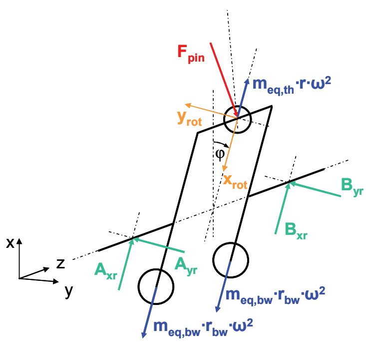


Figure 6. Acting forces on the crankshaft

Using the bending line method, the deformation of the crankshaft can be calculated. The general differential equation of the elastic line is

$$w''(z) = \frac{d^2 w(z)}{dz^2} = -\frac{M_b(z)}{E \cdot I} \tag{2}$$

w (z): deflection, M_b: bending moment, E: Young's modulus, I: area moment of inertia.

The deflection can be achieved by integrating the differential equation and adjusting the general solution to the boundary conditions.

The joint force F_{pin} has to be decomposed into a radial component and a tangential component. Each component leads to a bending moment of the crankshaft. The deflections in the associated spatial directions x_{rot} and y_{rot} can be calculated separately and superposed afterwards.

Having a look at the whole mechanism, especially the functional dimension piston position, it is possible to represent the deflection of the crankshaft like the clearance in section 4.3 by means of an additional vector in the functional relationship.

4.5 Integrated functional relationship for the piston position

Based on the calculation procedures of the previous sections, the functional relationship for the deviated piston position in the x-y-coordinate system shall be established.

At first, it is important to get an overview over the geometrical relations between the different coordinate systems which are necessary for the different calculations. They are shown on the left hand side in figure 7.

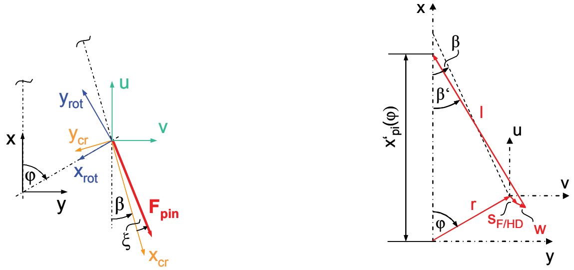


Figure 7. Left: Geometrical relations between the different coordinate systems, Right: Schematic illustration of the functional relationship of the piston position

The x-y-coordinate system is the global coordinate system which is used for the definition of the piston position. The spatial directions x_{cr} and y_{cr} are used for the calculation and representation of the joint force F_{pin}. In the coordinate system u-v, the hydrodynamic consideration of the conrod big end bearing takes place. The motion path is often illustrated in the x_{cr}-y_{cr}-system. The deflection of the crankshaft is considered in the directions x_{rot} and y_{rot}. Generally, x-y and u-v are fixed coordinate systems; x_{cr}-y_{cr} and x_{rot}-y_{rot} are moving.

The toleranced parameters for the following calculations are the link lengths r and l as well as the clearance s at the conrod big end bearing (NB: The deformation of the crankshaft is not a common toleranced parameter; it is a resulting geometrical deviation that occurs at operating state.)

The functional relationship is depicted on the right hand side in figure 7. The equation of the deviated piston position is

$$\begin{bmatrix} x'_{pi}(\varphi) \\ y'_{pi}(\varphi) \end{bmatrix} = \begin{bmatrix} r \cdot cos\,\varphi \\ r \cdot sin\,\varphi \end{bmatrix} + \begin{bmatrix} s_{F/HD,u} \\ s_{F/HD,v} \end{bmatrix} + \begin{bmatrix} w_u \\ w_v \end{bmatrix} + \begin{bmatrix} l \cdot cos\,\beta' \\ -l \cdot sin\,\beta' \end{bmatrix} \tag{3}$$

$s_{F/HD,u/v}$ is the displacement of the conrod relative to the crankpin at the joint with clearance. The indices indicate the calculation model (displacement in direction of the force or hydrodynamic consideration) and the spatial direction. w_u and w_v represent the deflection of the crankshaft.

Due to the displacement at the big end bearing and the deflection of the crankshaft in combination with the persisting joint condition between piston and housing (piston moves on the cylinder axis x), the angle β changes a little bit. The modified angle β' can be determined by using the joint constraint $y'_{pi}(\varphi) = 0$.

5 PERFORMED ANALYSES AND RESULTS

In this chapter, the analyses which were carried out based on the described approaches in chapter 4 are presented. The general calculation procedure is depicted in figure 8.

As already mentioned, the toleranced parameters are the

- crank radius r = 45 ± 0,02 mm
- conrod length l = 138 ± 0,05 mm and the
- clearance at the conrod big end bearing s = 0,06 ± 0,015 mm.

The link length deviations are inserted during the setup of the functional relationship. They are not taken into account in the initial kinematics which are the basis for the evaluation of the forces because the impact of these two deviations on the forces is negligible in this case. Based on the forces, the displacement at the big end bearing and the deflection of the crankshaft are evaluated separately and inserted into the functional relationship.

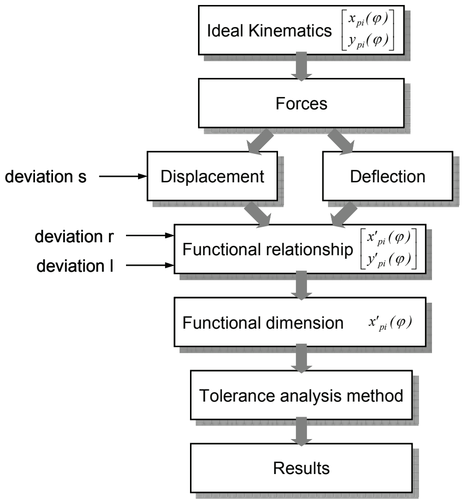


Figure 8. Calculation procedure

The following analyses can be divided in three sections:

1. Evaluation of differences between the two displacement models at the joint with clearance (displacement in direction of force vs. hydrodynamic consideration)
2. Arithmetical tolerance analysis of the piston position depending on the toleranced parameters and the deformation
3. Contributors analysis which gives information about the influence of the toleranced parameters on the deviation of the piston position

Due to the fact that the deviations are very small compared to the nominal dimensions, the results of the deviated piston position are displayed with respect to the ideal piston position (see equation 1).

Analysis 1

In Figure 9, the comparison of the two displacement models is illustrated.

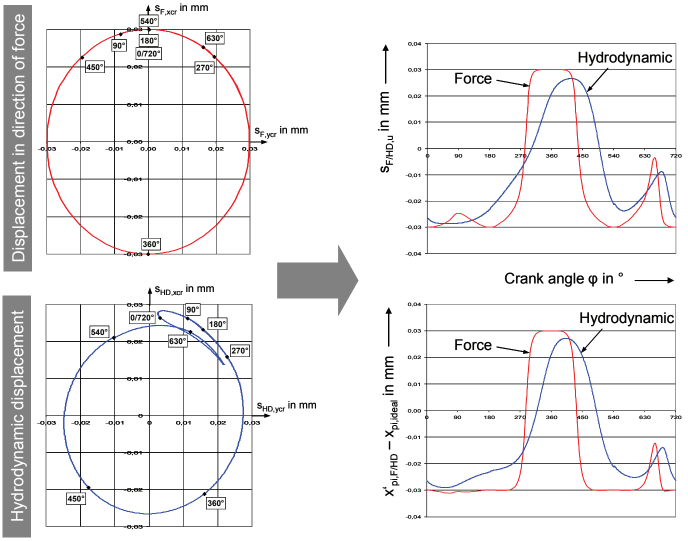


Figure 9. Comparison of different models for the displacement at the big end bearing

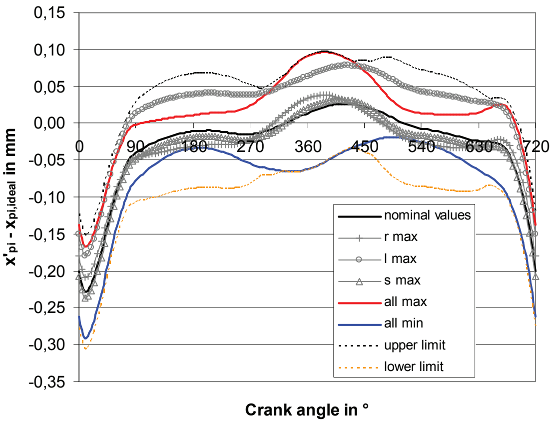
The clearance s in this case is 0,06 mm. The diagrams on the left hand side show the displacement of the conrod relative to the crankpin at the big end bearing for the whole motion cycle with respect to the x_{cr}-y_{cr}-coordinate system. On the right hand side, the displacements $s_{F,u}$ and $s_{HD,u}$ are compared for the u-v-coordinate system. Based on this, the deviated piston position (in the x-y-coordinate system) is evaluated. The adjustment of the angle β leads to the changes between the courses in the two diagrams on the right hand side. Basically, the courses of the displacements and the deviated piston positions of the force-based and the hydrodynamic approach have a similar shape. The problem is that they are shifted which leads to completely different values of the deviated piston position in some crank angle regions. The greatest difference occurs around 450°. For the following arithmetical considerations, the hydrodynamic model is chosen.

Analysis 2
The next step is the arithmetical tolerance analysis of the piston position. The aim is to receive information about the variation of the functional dimension for varying toleranced parameters. To get a first impression, the toleranced parameters are set to different values in different configurations (see table 1).

Table 1. Overview over different configurations of the toleranced parameters

	nominal	r max	l max	s max	all max	all min
r in mm	45	45,02	45	45	45,02	44,98
l in mm	138	138	138,05	138	138,05	137,95
s in mm	0,06	0,06	0,06	0,075	0,075	0,045

The deviated piston positions x'_{pi} for the defined configurations are shown in figure 10. The second diagram in figure 10 displays the arithmetical tolerance T_a of the piston position which can be calculated depending on the absolute values of the linearity coefficients and the tolerance intervals of the toleranced parameters (see [25]). The upper and lower limits of the piston position deviation are the results of the addition of $\pm T_a/2$ to the nominal value (NB: The intervals of the toleranced parameters are symmetric; consequently the nominal value is equal to the arithmetic mean). It can be seen that the deformation of the crankshaft has a dominating influence on the deviation of the piston position around φ=0°. In addition to this, the extension and the position of the tolerance interval of the functional dimension vary over the crank angle.



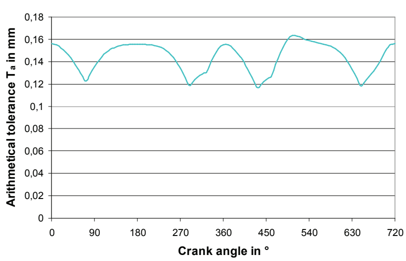


Figure 10. Left: Deviated piston positions for different configurations of the toleranced parameters; Right: Arithmetical tolerance T_a

Analysis 3
The goal of the third analysis is the quantification of the influence of each toleranced parameter on the variation of the functional dimension. For this purpose, a contributors analysis is carried out. It has to be kept in mind that the elastic deformation is not included in this calculation in the way that its con-

tribution is evaluated. It is only integrated via the functional relationship. This is due to the fact that the deformation is not a common toleranced parameter.
The contribution of a toleranced parameter in general is influenced by the functional relationship, by the range of the tolerance intervals and optionally by the manufacturing distributions of the toleranced parameters and the resulting distribution of the functional dimension. In the following, only the first two aspects are evaluated in an arithmetical contributors analysis.
The contribution of each toleranced parameter is calculated by multiplying the corresponding absolute value of the linearity coefficient with the tolerance interval of the toleranced parameter and dividing this by the arithmetical tolerance T_a of the functional dimension (see [25]). Figure 11 shows the linearity coefficient and the contribution of each toleranced parameter. The linearity coefficient reveals how much a geometrical change of the toleranced parameter affects the functional dimension just due to the geometrical relations in the mechanism. The (arithmetical) contribution additionally takes into account the range of the tolerance intervals.

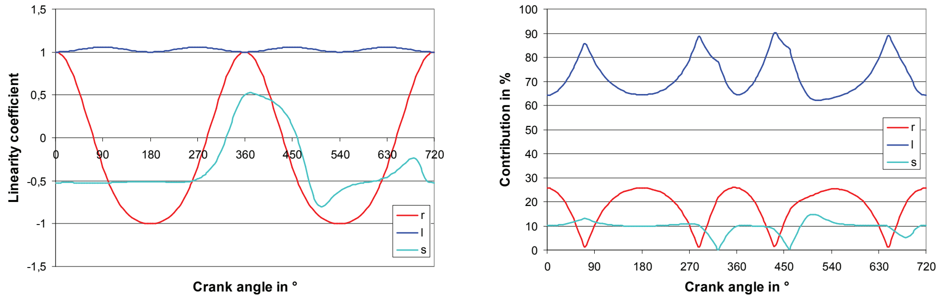


Figure 11. Left: Linearity coefficients for r, l and s; Right: Contributions of the toleranced parameters to the variation of the piston position

It can be observed that the linearity coefficients as well as the contributions vary over the crank angle. The conrod length l has a nearly constant value of approximately 1. This aspect in combination with the biggest tolerance interval leads to the dominating influence.
The results of the contributors analysis are required input information for tolerance reallocation. The idea is to tighten tolerance intervals of dominating toleranced parameters and at the same time to loosen tolerance intervals of toleranced parameters with low influence. However, tolerances can not be tightened in any order because this increases the manufacturing costs. So called tolerance-cost-curves additionally have to be taken into account during the tolerance optimization process. The specific problem of mechanisms is the variation of the contribution values. Comparing the situations at $\varphi = 0°$ and $\varphi = 75°$, order and values of the contributions differ which would lead to completely different reallocated tolerance values.

6 SUMMARY AND FUTURE PROSPECTS

In this paper, an integrated tolerance analysis approach for a crank mechanism inside a combustion engine is presented taking into account link length deviations, clearance at the conrod big end bearing and elastic deformation of the crankshaft. The clearance leads to a displacement of the conrod relative to the crankshaft. Two different models for the calculation of the displacement (in direction of force vs. hydrodynamic consideration) are compared. The deflection of the crankshaft is determined by the bending line method. Both effects – the clearance and the elastic deformation – are integrated into the tolerance analysis model by inserting additional vectors into the functional relationship. For the established functional relationship, an arithmetical tolerance analysis is carried out. The crank angle depending variation of the piston position is evaluated. In addition to this, the contributions of the toleranced parameters to the variation of the functional dimension are another analysis results. This approach enables the integrated tolerance analysis of different kinds of deviation for mechanisms which are subject to time-depending operating conditions and whose components have time- and load-depending positions. The calculation procedure is divided into different modules (e.g. module for the calculation of acting forces, module for the evaluation of the displacements at joints with clearance, module which comprises a certain tolerance analysis method). Calculation methods are often limited due to the fact that they act on certain assumptions. If more precise results with tightened assumptions are needed, the calculation methods inside the module can be exchanged. The interfaces to the adjacent modules persist. Generally, the approach can also be applied to other mechanisms because the basic representation of the motion behavior via vectors and matrices is not limited to the crank mechanism.

Up to now, the functional dimension was defined in the x-y-plane depending on a few toleranced parameters. Extending the plane to the three-dimensional space and considering more toleranced parameters would lead to a much more complex problem. For example, beside pure displacement of the involved components at joints with clearance, tilt can occur which influences as well the functional dimension.

The next step with regard to the application of tolerance analysis methods on the functional dimension would be the use of statistical tolerance analysis methods to be able to take into account manufacturing distributions of the toleranced parameters. In addition to this, a computer aided tolerance analysis process by coupling approved simulation tools for the different calculation modules shall be established. Therefore, it is important to analyze the interplay of product development and tolerance analysis process (e.g. the analysis process shown in this paper requires a huge amount of information about geometry and operation conditions, but this information is only defined in later phases of product development; consequently, the application of this approach is limited to theses phases). Another challenge in future will be to define and solve the optimization problem of tolerance reallocation against the background of varying contribution situations.

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