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PHOTOELASTIC RESEARCH OF THE HERTZIAN STRESS IN HCR-GEARS

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1. Introduction

High transverse contact ratio (HCR) involute gears (with transverse contact ratio $2<\varepsilon_\alpha<3$) have some advantages in comparison with standard involute gears ($\varepsilon_\alpha<2$), including bigger load capacity, more uniform torque transfer and reduced gear noise [Munro 1994, Murrell 1981, Villars 1999]. This is the result of double or triple teeths' pair contact, instead of single or double contact (Figure 1). Therefore, the use of HCR-gearing extends gear life. HCR-gears have bigger tooth addendum factor $h_{a0}*>1,25$ and normal pressure angle $\alpha_n \leq 20°$. The drawbacks of the HCR-gears are smaller tooth tip thickness, inconvenient sliding conditions between the teeth flanks, as well as higher accuracy required during manufacturing [Križan 2002, Križan 1999]. For all these reasons, doing some research on the load capacity of HCR involute gears could be useful.
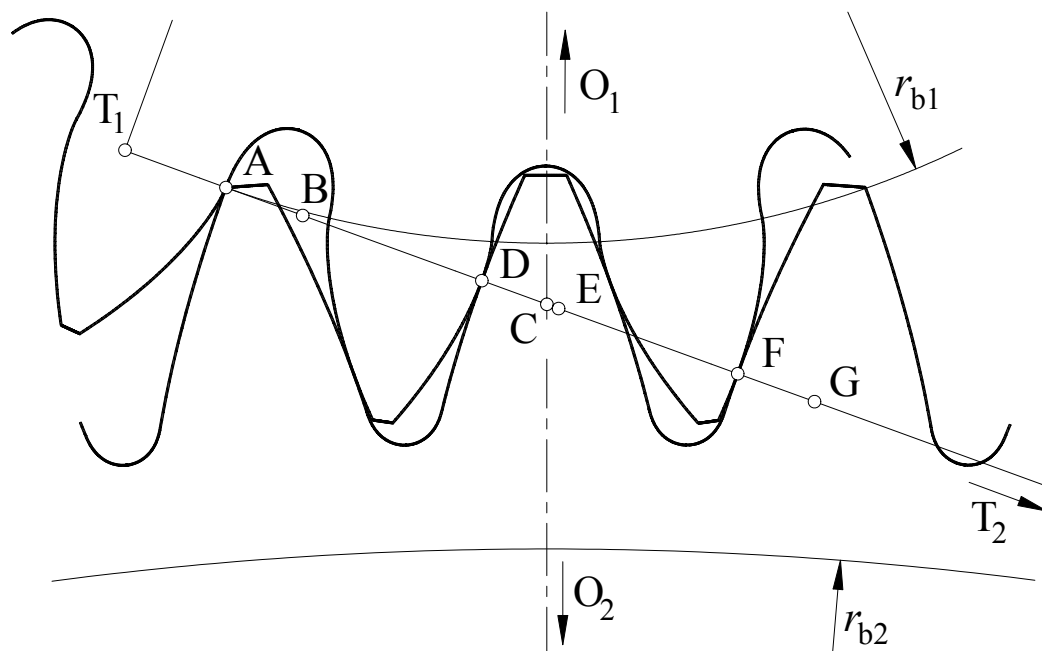

Figure 1. Gear tooth pair with a high transverse contact ratio (ε_α=2,3)

(A-B, D-E, F-G = triple contact; B-D, E-F = double contact)

2. Calculation of the force acting on tooth along the path of contact

Because of the double (two pairs of teeth) or triple (three pairs of teeth) contact between the teeth during the mesh of HCR-gears, every absolutely rigid and accurately machined tooth should be hypothetically loaded by alternatively 1/2 and 1/3 of the total normal force F_{bt} during the mesh (Fig. 2., dashed line ---). In reality the teeth are not rigid but elastic. The real load distribution for geometrically ideal shaped tooth during the mesh is in Fig. 2. represented by the continuous line (——). The real load distribution is calculated on the basis of tooth elastic deformation using the computer program developed for this purpose [Lovrin 2001]. An example is shown in Fig. 2: the maximal normal force F_{bt} on a tooth in this case is equal to 62% of the total force between the pinion and the gear. In some cases, depending on the actual toothing geometry and manufacturing accuracy, the force acting on the tooth can be even bigger, up to approximately 70% of the total force. On the basis of the real load distribution it is possible to precisely calculate the real Hertzian stress along the path of contact.
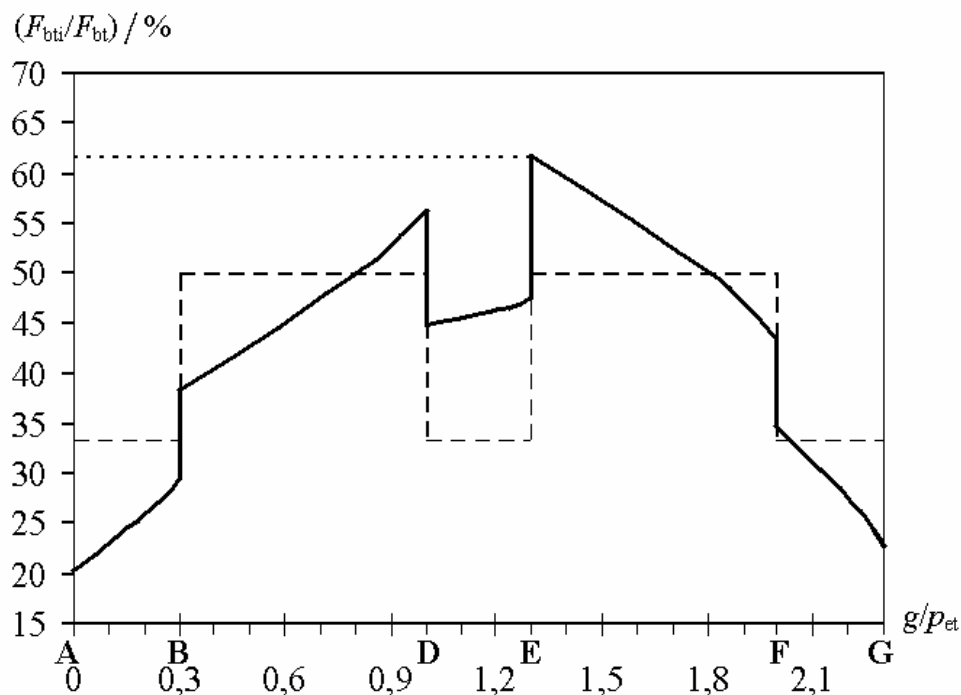

Figure 2. Hypothetical (---) and the real load distribution (——) on HCR-gears teeth along the meshing line (ε_α=2,3)

3. Photoelastic model

The usual procedure for photoelastic stress analysis in standard involute teeth with $\varepsilon_\alpha < 2$ is to make the model of a gear wheel segment with about three teeth, and then to apply the load on the middle-tooth. More complicated method is to make the models of two gear segments and to mesh them together.

The situation with the stress analysis in HCR-gears is even more complicated because of the double or triple contact. The tooth stresses depend on the tooth form, which is notorious, but also on the actual load acting on the tooth. This load notably changes its amount during the mesh period of the gears (Fig. 2). So, the photoelastic model must have three pairs of teeth in contact, which means that the model has to be manufactured very accurately. The deviations from the theoretical profile would significantly influence the load distribution on teeth pairs, and hence the stress in the tooth.

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Therefore, the two-dimensional models of HCR-gears in mesh were manufactured on a very precise numerically controlled high speed milling machine. In order to avoid the edge effect, the milling was carried out gradually, the last machined layer being only 0,1 mm.

The experimental model has two segments with three teeth each. The gear segments (6) and (2) are fixed in a loading frame (3), (Fig. 3). The load is introduced by means of a hand-wheel (4) and a spindle (5), which can move only axially thanks to the guideway (7), thus applying the force on the extension piece (9) of the gear segment (6). The applied force can be read on the ring dynamometer (8). The total force between the gear segments can be calculated by taking in account the radii r_1 and r_d.

To enable the double/tripple contact between the segments in different positions, the possibility of model shifting has been foreseen. The segment (2) can be fixed in the loading frame (3) on different heights by means of tightening plates (1) and bolts. The segment (6) can rotate on the eccentric axle in the ball bearing (10), thus enabling a precise adjustment of the gear centre distance a.

The parameters of the model-gears were: normal pressure angle $\alpha_p = 20°$; module $m_n = 22$ mm; tooth addendum factors $h_a*_{01/2} = 1,5$; tip radius of the cutter tool factor $\rho_a*_{01/2} = 0,341$; bottom clearence factor $c*_{1/2} = 0,1$; model thickness (gear width) $b_{1,2} = 10$ mm; teeth number $z_{1,2} = 22/88$; addendum modification coefficients $x_{1,2} = 0$; tooth tip diameter $d_{a1/2} = 545,6 / 1997,6$ mm; center distance $a = 1210$ mm; transverse contact ratio $\varepsilon_{\alpha t} = 2,30$.
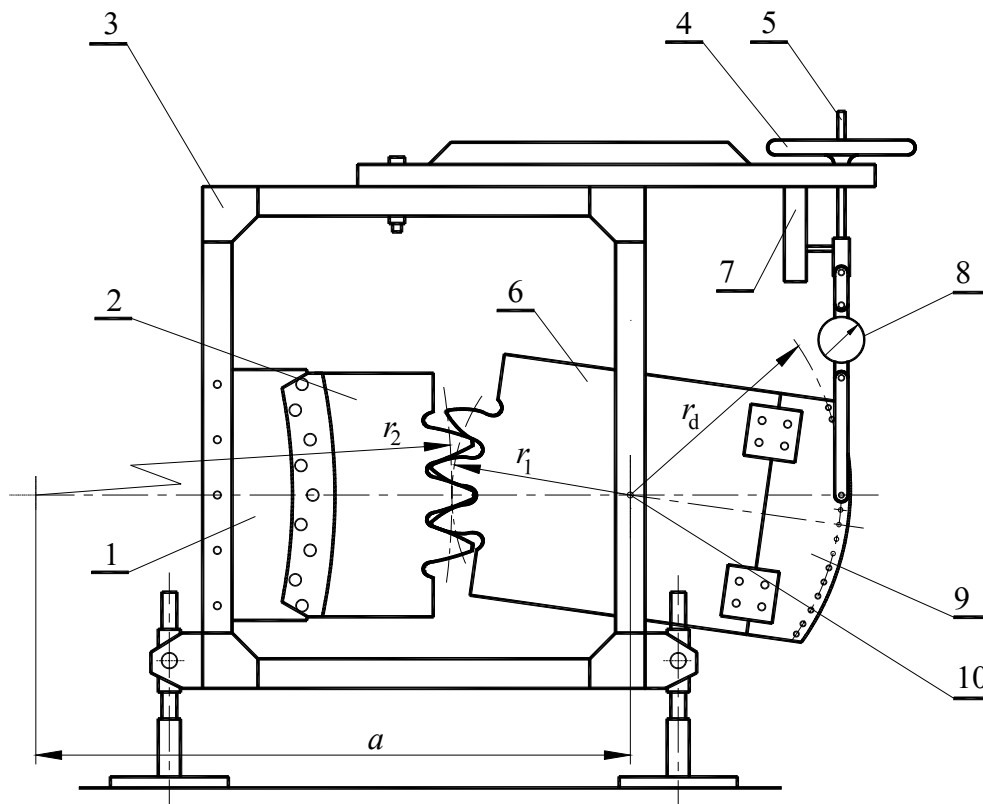

Figure 3. Loading frame for HCR-gears models

4. Analysis of the Hertzian stress in HCR-gears

One-hundred and eight previously taken photographs (an example in Fig. 4) of loaded HCR-gears photoelastic models were analysed. The Hertzian stresses have been obtained for a 22 different positions of meshing gears during the contact (i.e. from the point A to the point G on the meshing line)

using various applied loads. In this paper the results are presented for the total tangential force $F_t = 1798$ N and the total normal force on the meshing line $F_{bt} = 1913$ N.

The experimentally obtained stresses were compared with the values based on the determination of the real load distribution [Lovrin, 2001] (Fig. 2).

Figure 4. Photograph of the loaded HCR-gears photoelastic model in triple contact
($F_{bt} = 1913$ N)

The realistic Hertzian (nominal contact) stress along the path of contact has been calculated using the following Hertz's equation:

$$\sigma_{H0i} = \sqrt{\frac{F_{bti} \cdot E}{2 \cdot \pi \cdot \rho_{ri} \cdot b_g \cdot (1 - \nu^2)}} \tag{1}$$

where F_{bti} = the realistic normal force (acc. to Figure 2) in the corresponding point i along the path of contact A-G, E = modulus of elasticity, ρ_{ri} = reduced radius of the curvature of the teeth flanks in the point i, b = mutual active gear width and ν = Poisson's ratio.

Nominal contact (Hertzian) stress, in the corresponding points E for pinion and D for wheel, according to [DIN 3990, 1987], have been calculated using the equation with included following Z-factors

$$\sigma_{H01,2} = Z_H \cdot Z_E \cdot Z_{B,D} \cdot Z_\beta \cdot \sqrt{\frac{F_t}{d_1 \cdot b} \cdot \frac{u+1}{u}} \cdot Z_\varepsilon \tag{2}$$

where Z_H = zone factor, Z_E = elasticity factor, $Z_{B,D}$ = pinion resp. wheel single tooth contact factor, F_t = nominal tangential force on reference cylinder (total tangential force), u = transmission ratio, d_1 = pinion reference diameter and Z_ε = contact ratio factor.

The Figure 5 shows Hertzian stresses along the meshing line for pinion and wheel in the photoelastic model of the HCR gear pair, calculated using the own developed software [Lovrin, 2001] and equation (1), represented by the continuous line (━).The experimentally obtained Hertzian stresses are represented by an asterisk (∗). Hertzian stresses are also presented, for the pinion in the point   E (○) and the wheel in the point D (●), calculated according to the DIN-Standard procedure [DIN 3990, 1987] and using the appropriate equation (2).
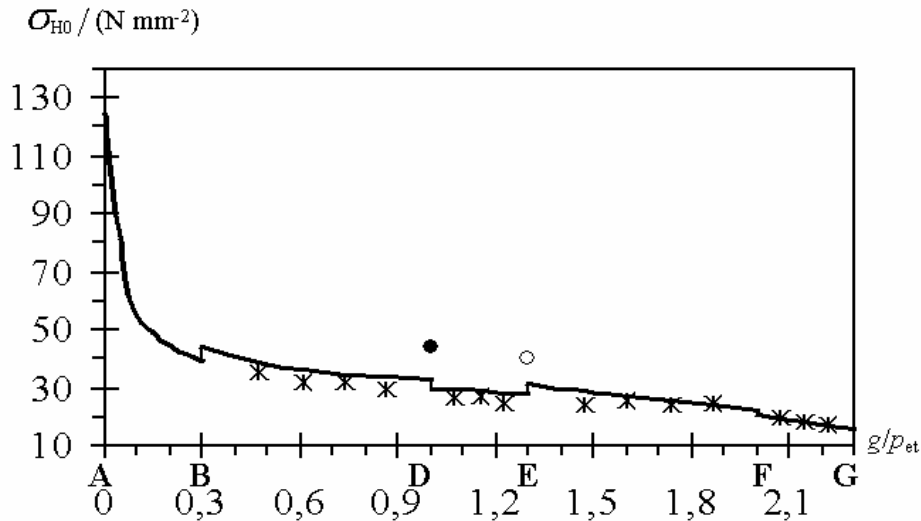

Figure 5.   Hertzian stress along the meshing line in the HCR-gears model (F_{bt} = 1913 N)

(━━ (1) [Lovrin, 2001], ∗ experimentally, ○ pinion and ● wheel (2) [DIN 3990, 1987])

An example in the Figure 5 shows very good concordance between the experimental results and the results of the numerical approach.

The photoelastic research of the Hertzian stress in HCR-gears [Lovrin, 2001] and the above presented results point at a general conclusion that the Hertzian stresses in HCR-gears are smaller than the results obtained using the DIN 3990 procedure.

The photoelastic research of the tooth root stresses in HCR-gears [Lovrin, 2000] shows that DIN-Standard procedure produces too intense stresses in the tooth root too.

It can be said that the DIN 3990 Standard calculation gives rather inaccurate results, with higher stresses than they really are. This is because the DIN-Standard HCR-gears calculation procedure is based on the presumption, i.e. approximation, that there is a total force Fbt acting on the tooth, although hypothetically it can be at the most 1/2 of the total force. This approximation has been accepted in order to enable the use of standard formulae (for standard toothing with εα<2) for the Hertzian and tooth root stress calculation of the HCR-gears.

5. Conclusion

By using a photoelastic model with double and triple teeth contact, it was proven that there is a good concordance between the experimental results and the numerical calculations. A comparison of experimental results with the values that are obtained by using the DIN-Standard HCR-gears calculation and the results obtained by the numerical procedure [Lovrin, 2001], shows that DIN-Standard calculation procedure produces too intense Hertzian stresses, which means that these gears will be oversized.

Our own developed method for a more precise Hertzian stress determination, using load distribution on gear tooth pairs in mesh calculated on the basis of their elastic deformations [Lovrin, 2001], enables a more reliable calculation of HCR-gears load capacity. Thus HCR-gearboxes designed by use of the presented improved calculation will not be oversized i.e. will be smaller and lighter.

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