

## SYSTEMATIZATION OF DESIGN PROBLEMS BY GRAPH THEORY

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### 1. Introduction

The primary purpose of Japanese Fast Breeder Prototype Reactor(FBR) 'MONJU' is to obtain technical information from the design, construction and operation, and to apply this information to future designs and construction of safe, economic, and competent, fast breeder reactors. Currently available engineering documents and database of 'MONJU' are divided into engineering fields, e.g., neutron kinetics, thermal-hydraulics, high-temperature structures, electricity, etc. or into systems/components, e.g., reactor vessel, pumps, steam generators, etc.

The design of large-scale systems like 'MONJU' that secures both safety and reliability requires another area of knowledge that requires the organizing and sequencing of a huge number of decision activities to determine each of the specifications based on the type of engineering information mentioned above. This type of organization and sequencing of decision activities in designing 'MONJU' is currently remembered mostly by the original designers of 'MONJU', who are now retiring or leaving the FBR community. Thus systematization of design problems is critical not only for frequently used industrial products [Eppinger 2001], but also for rarely designed, complex, and safety critical nuclear power plants.

The authors propose a graph-theory-based method to systematize design problems by representing a design problem as a bipartite graph consisting of a set of design conditions and a set of specifications. This method is basically equivalent to Design Structure Matrix(DSM) approach[Steward], but the authors believe that this graph-theory based approach have some advantages in capturing the structure of the given design problems.

### 2. Outline of the proposed method

The proposed method is based on a representative scheme of a design problem as a bipartite graph from a set of design conditions to another set of specifications. This method is explained with a small hydraulic cylinder model shown below in Fig. 1. Suppose that the design problems of this model consists of 6 design conditions and 10 specifications as correlated in Tab. 1. The designer is not necessarily requested to judge every design condition to be met, nor to satisfy every design condition by adjusting the values of involved specifications. It is expected to defer this kind of judgment or adjustment to experts, specialists, or existing analytical tools. This research focuses on how to represent decision making sequences with minimal difficulty

If the design of a hydraulic cylinder with user-specified values (e.g. permissible stress, guaranteed number of strokes, inner pressure, and weight), the design problem could be represented as a bipartite graph based on the new set of 10 design conditions (6 old + 4 new) and the set of 10 specifications as shown in Fig. 2.

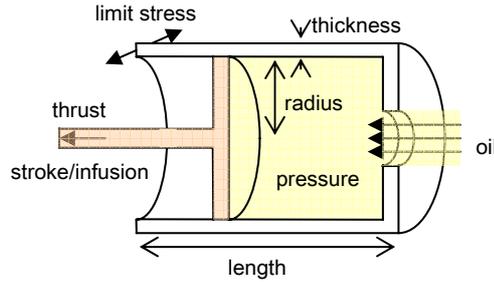


Figure 1. Hydraulic Cylinder Model

Table 1. Design conditions and specifications of the cylinder model

conditions	specification involvements	satisfaction criteria
Piston movement	$\text{movement/infusion} = 1/(\pi * \text{radius}^2)$	calculation
Cylinder integrity	$\text{loaded stress} = \text{inner pressure} / (2 * \text{radius} * \text{thickness}) < \text{permissible stress}$	calculation
Thrust	$\text{Thrust} = \text{inner pressure} * \pi * \text{radius}^2$	calculation
Weight □ Size	$\text{radius}^2 * \pi * \text{length} * \text{thickness} * \text{density} = \text{weight}$	calculation
Oil leakage	probable by a function of (radius, thickness, length, movement, strokes, precision)	experiment by experts
Fabricability	function of (radius, thickness, piston movement, precision)	fabricator's judgment

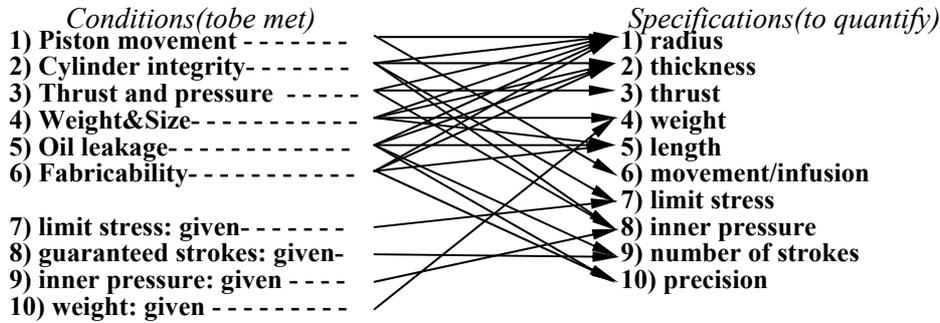


Figure 2. A bipartite graph representing the hydraulic cylinder design problem

The strictest constraints in this design problem are conditions 7) through 10), because these conditions are dependent on only 1 specification. (Note that conditions other than the user-request can also be in general the most important.) Therefore values of specification 7), 9), 8) and 4) are the first to be assigned. Other than these conditions and specifications, there remains no condition dependent on only 1 specification. In this case the set of strictest constraints on the remaining conditions are selected so that:

- All the constraints within the set can be met only by iterative value adjustments of the relevant specifications. If the conditions and specifications were equations and variables that cannot be determined individually then, this set would represent a matrix that cannot be broken down.
- No condition within the set is dependent on conditions outside the set.

Regarding the cylinder model the second most important set consists of conditions 2), 4), 5) and 6), and specifications 1), 2), 5), 6) and 10). After removing this set, the remaining part of the problem only consists of 2 arrows from condition 1) to specification 6), and from condition 3) to specification 3), respectively.

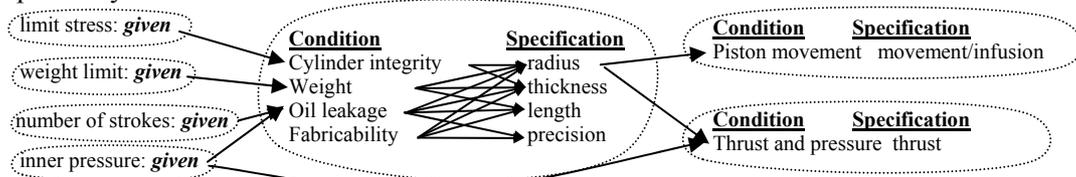


Figure 3. Derived decision making sequence for the cylinder design problem

Thus derived decision making sequence is diagrammatically shown in Fig. 3.

This procedure for decomposition and ordering of a bipartite graph is defined equivalently as "Dulmage-Mendelsohn Decomposition" (referred to hereafter as "DM decomposition") [Dulmage 62].

Note that priorities among decision making steps are in the opposite direction when processed by the 'as-defined' DM decomposition.

### 3. Application of the proposed method to FBR design

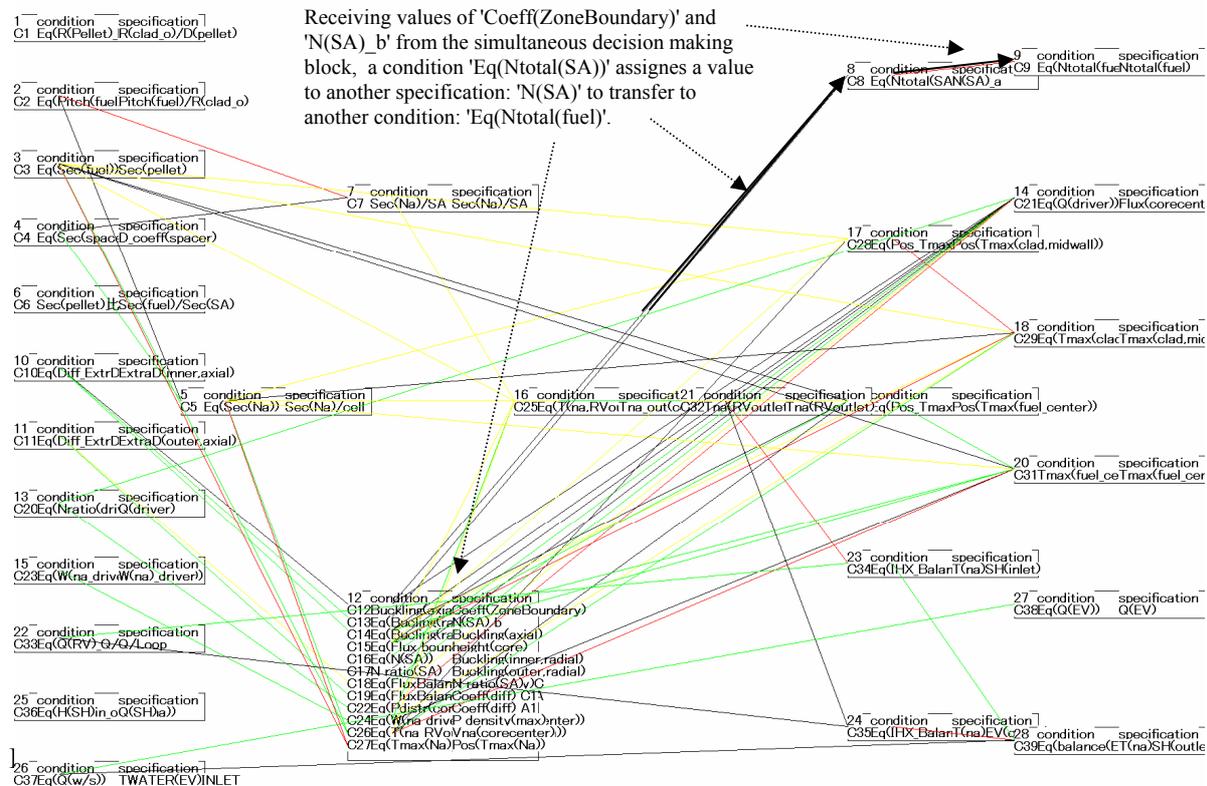
#### 3.1 Validation by a simplified model of FBR 'MONJU'

Frequent concepts on possible design deviations are expected to be effective for carrying over accumulated engineering knowledge at 'MONJU'. A software program "Plant Planning Design System" is being written [Kasagawa 2002], to derive entire design deviations based on assumed change of particular specifications. Currently this system includes equations and variations relevant to core neutron kinetics, balance of heat generation and coolant flow in the core, and heat balance of the heat transport systems (Tab. 2). The calculation algorithm was built by design experts who were involved with the original design of 'MONJU'.

The prototype of the graph-theory based software was validated by taking the equations and variations of "Plant Planning Design System" as the design conditions and specifications, respectively and by confirming that the design problem is decomposed and arranged equivalently to the algorithm employed in "Plant Planning Design System", as shown in Fig. 4.

**Table 2. Conditions and specification examples of the simplified 'MONJU' model**

category	specifications
core neutron kinetics	21 design conditions and 42 specifications: core radius, core length, pellet diameter, sheath thickness, fuel pin pitch, fuel pin number per assembly, Pu enrichment, etc.
in-core thermal balance	11 design conditions and 30 specifications: core heat generation, driver area heat generation, maximum axial power density, maximum fuel center temperature, maximum coolant temperature, etc.
plant heat balance	7 design conditions and 15 specifications: reactor inlet temperature, primary sodium flow rate, IHX heat transfer area, main steam condition, transported heat per loop, etc.



**Figure 4. Derived design sequence of the simplified 'MONJU' model**

Each rectangle in Fig. 4 stands for a decision making step where conditions to be met and specifications to quantify are listed on the left hand side and the right hand side of each rectangle,

respectively. Rectangles with plural conditions/specifications mean simultaneous problems. Rectangles on the left periphery of Fig. 4 correspond to the given conditions or user input variables set in the "Plant Planning Design System". Values of specifications assigned in each rectangle are transferred by lines drawn from the rectangle to the immediate right, where subsequent decisions of specification values are progressed. Application of graph theory enables handling of design problems with a greater number of specifications than the number of design conditions.

### 3.2 Design procedure revisions by adding a new condition

New design conditions may appear in seeking for more rationalized plant concepts in order to securely avoid adverse phenomena with narrowed-down margins among specifications. If the designers keep following empirically proven decision making sequences even after this discovery, the specification assignment procedures tend to be inefficient and unreliable, because the new condition that avoids the adverse phenomenon is likely to be left unconsidered until the entire empirical design procedure is finished. If this finally considered condition is found not to be satisfied, the designers may find it difficult to determine where to restart the design process.

The DM-decomposition is effective in this kind of situation. Suppose that a design team is coping with a device which used to be represented by a bipartite graph shown in Fig. 5. The specification assigning sequence may be as shown in Fig. 6.

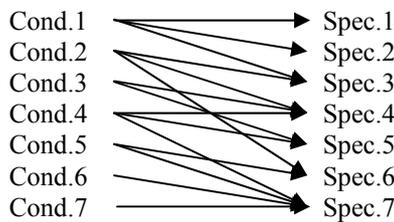


Figure 5. Original design problem

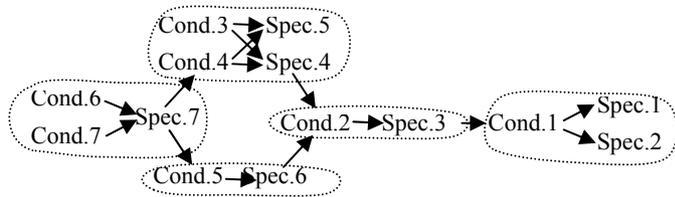


Figure 6. Original specification assignment procedure

If on completion of the design problem a further condition involving spec 2, 4&6 requires to be analysed Fig 7. there is no requirement to repeat the complete iterative process. The DM decomposition can give a new design sequence as shown in Fig 8. This sequence is simpler to process as there are minimal steps of decisions to process.

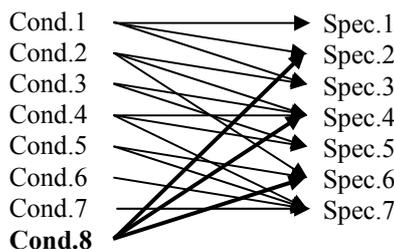


Figure 7. Newly recognized design

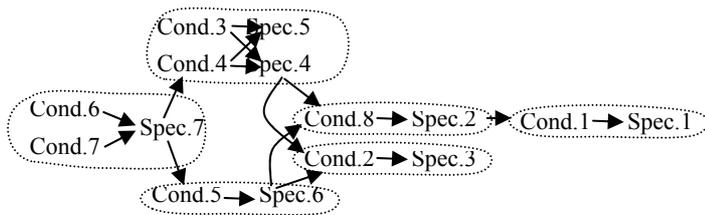


Figure 8. Revised assignment procedure

## 4. Representation of Non-quantitative specifications

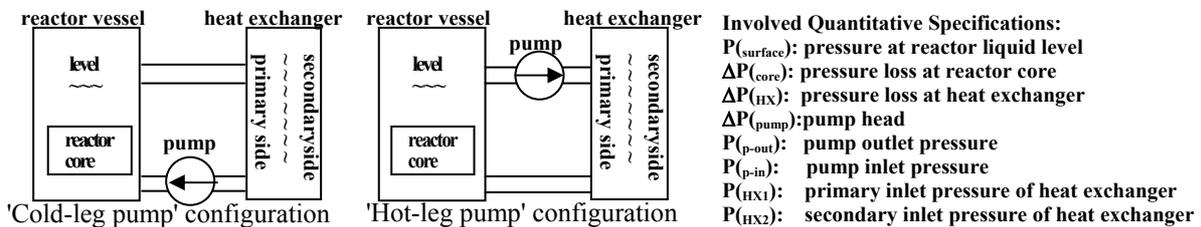
Tab. 3. shows the main features of 'MONJU', consisting of non-quantitative and quantitative specifications. There are many cases that quantitative specifications are dependent of a selected option of a non-quantitative specification. A quantitative specification term 'Reactor Outlet Coolant Temperature' exists because the selected option for a non-quantitative specification 'reactor type' is 'Loop'. Also designed nominal/maximum temperatures and pressures are dependent on selected materials for structures and fluids. Thus non-quantitative specifications have the greater importance.

Some kind of non-quantitative specifications can be handled in the bipartite graph representation of design problems, as explained below.

Generated heat in a nuclear power plant is transported from the reactor core by circulating coolant and transferred to the adjacent coolant circulation loop through a heat exchanger. There are two options for the location of the circulation pump which can either be located on the inlet side of the reactor vessel (termed a “cold leg pump”) or on the outlet side of the reactor vessel (termed a “hot leg pump”) as shown in Fig 9.

**Table 3. Important Specifications of 'MONJU'**

Specification Item	Selected Option for 'MONJU'
<b>20 Non-Quantitative Specifications</b> Reactor type Reactor Liquid Level Maintenance Scheme Pump Locations Decay Heat Removal Fuel Type Fuel Pin Sheath Material Core Configuration Core Support Structure Coolant Material of Heat Transport System Reactor Coolant Penetration Scheme Countermeasure against Thermal Stratification Intermediate Heat Exchanger Type Steam Generator Type In-Vessel Fuel Shuffling Scheme Fuel Transport to/from Containment Vessel Storage Scheme of Decaying Spent Fuel Cooling Down Scheme of Spent Fuel Reactor Containment Reactor Main Building	Loop (The other option is “Tank”) Elevated Horizontal Piping and Low Component Guarding Cold Leg Air Cooled Heat Exchanger Installed Parallel with Steam Generators Mixed Oxide of Plutonium and Uranium SUS316 Equivalent Radial Location of Two Homogeneous Regions Suspended Core Side Liquid Sodium SUS304 Lower Inlet and Upper Outlet In-bucket Twin level Control Vertical Counter Flow Heat Exchanger by Straight Tubes w/o Cover Gas Once Through Divided Components with Helical Coil Heat Transfer Tubes Subassembly Handling Arm Fixed on Single Rotation Plug Mobile Fuel Transporter Ex-Vessel Fuel Storage Tank Sealed Can with Filled Water in Water Pool Cylindrical Carbon Steel with Top and Bottom Semi-spheres Cement Reinforced Steel Wires Fixed to Rock Foundation
<b>13 Quantitative</b> Generated Electricity Control Rods Maximum Fuel Pin Sheath Temperature Fuel Contents and Pu Enrichment Diameter of Fuel Pin Sheath Number of Fuel Elements per Subassembly Maximum Axial Power Density Average Burn-up of Withdrawn Subassembly Core Size(Height/Diameter) Breeding Ratio Number of Heat Transport Loops Reactor Outlet Coolant Temperature Steam Condition at Turbine Inlet	280 Megawatts 13 Power Controllers and 6 Backup Stoppers 675°C 15% at Inner Core and 20% at Outer Core 6.5 mm 169 360 W/cm ~ 80,000MW*day/ton 0.93m/1.8m ~ 1.2 3 529°C 127Kg/cm <sup>2</sup> , 483°C



**Figure 9. Two options for circulation pump location**

Both configurations have exactly the same set of quantitative specifications. There are six design conditions among these quantitative specifications, three of which are common and the others are dependent on configurations, as shown in Tab. 4. The pressure loss of the reactor core is naturally assumed to have little margin for adjustment because of tight conditions of thermal and neutron-kinetic constraints. Thus the resulting bipartite graphs and DM-decomposed graphs for these configurations are shown in Fig. 10.

**Table 4. Condition-specification relations of the two configurations**

design conditions	cold-leg pump	hot-leg pump
1) Pump head=pressure loss	$\Delta P_{(numn)} = \Delta P_{(core)} + \Delta P_{(HX)}$	
2) Cavitation prevention	$P_{(n-in)} > \text{Cavitation Limit Pressure}$	
3) $P(1rv) < P(2rv)$	$P_{(HX1)} < P_{(HX2)}$	
4) PumpOutlet-LiquidLevel	$P_{(n-out)} = P_{(level)} + \Delta P_{(core)}$	$P_{(n-out)} = P_{(level)} + \Delta P_{(core)} + \Delta P_{(HX)}$
5) PumpInlet-LiquidLevel	$P_{(n-in)} = P_{(level)} - \Delta P_{(HX)}$	$P_{(n-in)} = P_{(level)}$
6) EX1ryInlet-LiquidLevel	$P_{(HX1)} = P_{(level)}$	$P_{(HX1)} = P_{(level)} + \Delta P_{(numn)}$

These figures show that options of a non-quantitative specification determine an inter-dependency of relevant quantitative specifications. The proposed graph-theory based method has a potential for handling non-quantitative specifications automatically alternating the structures.

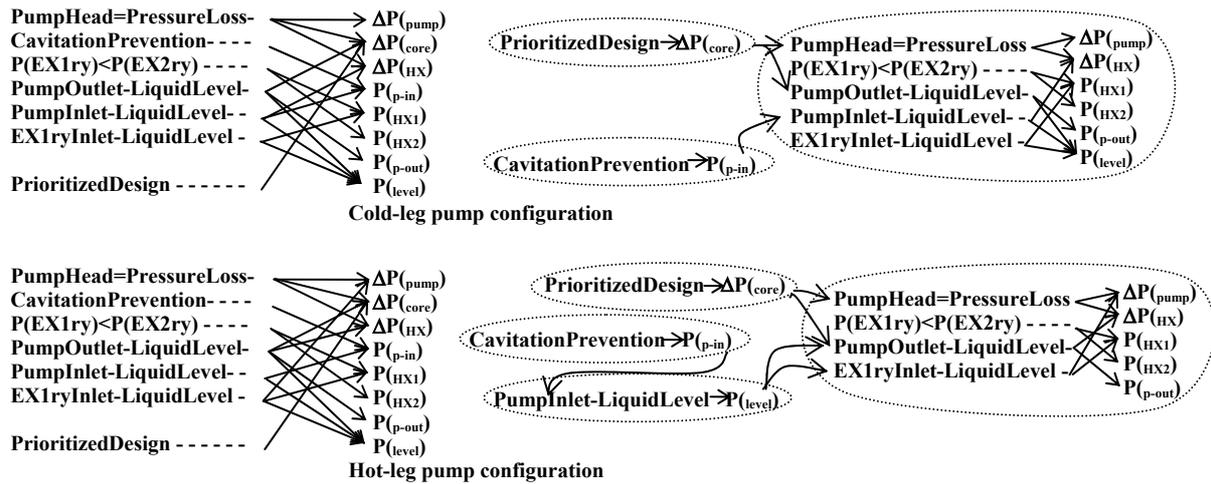


Figure 10. The bipartite graphs and DM-decomposed graphs for the 2 configuratio

## 5. Open Problems

The resulting compromises made in the resolution of conflicting design conditions or requirements, are in general mitigated by safety margins, (e.g., inner pressure is limited by the conservative assignment of permissible stress of the cylinder dimensions and material). Ideally, the probability of an adverse event (e.g., cylinder rupture) is diminished by properly assigning the proposed safety margin. Therefore, safety margin distribution in such situations is important in the design of large-scale systems. Otherwise, sequenced adverse phenomena may result in an incident involving the entire system. To optimize the entire system reliability, the proposed method should be combined with the techniques for probabilistic safety assessment.

## 6. Conclusion

A graph-theory-based systematization method of design problems has been proposed aiming at carrying over obtained engineering knowledge in Prototype Fast Breeder Reactor 'MONJU'. This method focuses on deriving decision making sequence in assigning values on specifications, rather than pursuing accuracy in each judgment or evaluation. A prototype software has been verified by comparing the derived decision making sequences from a set of equations among major specification parameters of the 'MONJU' plant to an empirically built algorithm by human experts actually involved in designing 'MONJU'. This method has a potential in handling non-quantitative specifications by automatically switching inter-dependency structure among the relevant quantitative specifications.

## References

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